Improved Bounds on Permutation Arrays for Chebyshev Metric

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Let σ and π be two permutations over an alphabet $\Sigma \subseteq [n] = \{1, 2, \ldots, n\}$. The Chebyshev distance between σ and π, denoted by $d(\sigma, \pi)$, is $\max\{\ |\sigma(i) - \pi(i)|\}$. For an array (set) A of permutations (strings), the pairwise Chebyshev distance of A, denoted by $d(A)$, is $\min\{\ d(\sigma, \pi) \mid \sigma, \pi \in A, \sigma \neq \pi\ \}$. An array A of permutations on $[n]$ with $d(A) = d$ will be called an (n, d)-PA. Let $P(n, d)$ denote the maximum cardinality of any (n, d)-PA. We prove new lower and upper bounds on $P(n, d)$. One of the methods for finding new (n, d)-PAs is based on prefixes and suffixes of permutations. It introduces two new problems.
1) For $m < n$, find the largest set A of subsets of $[n]$ of size m with $d(A) \geq d$.
2) For an alphabet $\Sigma \subseteq [n]$, find the largest set B of permutations on Σ with $d(B) \geq d$.

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