Graphs that allow two distinct eigenvalues

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Let G be a connected graph on n vertices and let $\mathcal{S}(G)$ denote the set of all real symmetric $n \times n$ matrices $A = [a_{ij}]$ such that $a_{ij} = 0$ if and only if $\{i, j\}$ is not an edge of G. The diagonal entries of A can take any value. The inverse eigenvalue problem of a graph asks to determine all possible spectra of matrices in $\mathcal{S}(G)$. A fundamental subproblem is to determine the minimum number of distinct eigenvalues over all matrices in $\mathcal{S}(G)$. This parameter is denoted by $q(G)$. For example $q(G) = n$ if and only if $G = P_n$, the path on n vertices. The graphs with $q(G) = n - 1$ have also been characterized. Determining those graphs with $q(G) = 2$ has been much more difficult. A recent advance has been to determine the minimum number of edges in a graph G with $q(G) = 2$. The graph G must have at least $2n - 3$ edges if n is odd and at least $2n - 4$ edges if n is even. The graphs for which equality is attained are characterized.