Probing Dot Product Graphs

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An undirected graph $G = (V, E)$ is a probe $\mathcal{C}$ graph if its vertex set can be partitioned into two sets, N (nonprobes) and P (probes), where N is independent and there exists $E' \subset N \times N$ such that $G' = (V, E \cup E')$ is a $\mathcal{C}$ graph. A *dot product graph* is a graph G such that there exists a function $f : V(G) \to \mathbb{R}^k$ such that, for $x, y \in V(G)$, $xy \in E$ if and only if $f(x)^T f(y) \geq 1$. The minimum k for which such a function exists for G is the *dot product dimension of* G, denoted $\rho(G)$.
Structural characterizations of dot product graphs for some fixed k, and determining $\rho(G)$ for G which are $\mathcal{C}$ graphs are two problems on which I focus and to these ends we explain *k dot product probe graphs*. I characterize 1-dot product probe graphs and discuss how a 2-SAT function for identifying them.

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