

Title: Generalized Commutation in Semigroups

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To generalize commutation in a semigroup  $S$ , we say that elements  $u, v \in S$  *contextually commute* if there exist  $x, y \in S^1$  such that  $xuvy = xvuy$ . It is straightforward that all elements contextually commute in many noncancellative semigroups, raising the question of whether the phenomenon is purely a byproduct of noncancellation. To test this, we seek a class of semigroups for which contextual commutation and noncancellation coincide exactly. More precisely, we define a *purely contextually commutative* (PCC) semigroup to be one in which for any two elements  $a, b \in S$ , there exist  $x, y \in S^1$  such that  $xay = xby$  iff there exist  $u, v \in S^1$  such that  $a = uv$  and  $b = vu$ . Examples include rectangular bands, cancellative commutative semigroups, and direct products of PCC semigroups. Others include the bicyclic monoid, the plactic monoid of rank 2, and a semigroup of tropical matrices. We show that a completely simple semigroup is PCC iff it is a rectangular abelian group.

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