Title: Generalized Commutation in Semigroups

Author: Peter Alspaugh*, University of South Florida
Joint work with Natasha Jonoska, University of South Florida

To generalize commutation in a semigroup S, we say that elements $u, v \in S$ *contextually commute* if there exist $x, y \in S^1$ such that $xuvy = xvuy$. It is straightforward that all elements contextually commute in many noncancellative semigroups, raising the question of whether the phenomenon is purely a byproduct of noncancellation. To test this, we seek a class of semigroups for which contextual commutation and noncancellation coincide exactly. More precisely, we define a *purely contextually commutative* (PCC) semigroup to be one in which for any two elements $a, b \in S$, there exist $x, y \in S^1$ such that $xay = xby$ iff there exist $u, v \in S^1$ such that $a = uv$ and $b = vu$. Examples include rectangular bands, cancellative commutative semigroups, and direct products of PCC semigroups. Others include the bicyclic monoid, the plactic monoid of rank 2, and a semigroup of tropical matrices. We show that a completely simple semigroup is PCC iff it is a rectangular abelian group.

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