On Hamilton cycle decompositions of complete multipartite graphs which are both cyclic and symmetric

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Let G be a graph with v vertices. A Hamilton cycle of a graph is a collection of edges which creates a cycle using every vertex. A Hamilton cycle decomposition C of G is a partition of the edges of G into Hamilton cycles. A Hamilton cycle decomposition is cyclic if the set of cycle is invariant under a full length permutation of the vertex set. We say a decomposition is symmetric if all the cycles are invariant under an appropriate power of the full length permutation. Such decomposition are known to exist for complete graphs and families of other graphs.

In this work, we show the existence of cyclic *n*-symmetric Hamilton cycle decomposition of a family of graphs, the complete multipartite graph $K_{m \times n}$ where the number of parts, *m*, is odd and the part size, *n*, is also odd. We classify the existence where *m* is prime and prove the existence in additional cases where *m* is a composite odd integer.

Keywords: complete multipartite graph, Hamilton cycle decomposition, cyclic & symmetric decomposition