Almost linear kernels for generalized covering and packing on sparse graphs

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Let $\mathcal{F}$ be a family of graphs and let p, r be nonnegative integers. For a graph G and an integer k, the $(p, r, \mathcal{F})$-Covering problem asks whether there is a set $D \subseteq V(G)$ of size at most k such that if the p-th power of G has an induced subgraph isomorphic to a graph in $\mathcal{F}$, then it is at distance at most r from D. The $(p, r, \mathcal{F})$-Packing problem asks whether the p-th power of G has k induced subgraphs $H_1, \ldots, H_k$ such that each H_i is isomorphic to a graph in $\mathcal{F}$, and for distinct $i, j \in \{1, \ldots, k\}$, the distance between $V(H_i)$ and $V(H_j)$ in G is larger than r.

We show that for every fixed nonnegative integer r and every fixed nonempty finite family $\mathcal{F}$ of connected graphs, $(p, r, \mathcal{F})$-Covering with $p \leq 2r + 1$ and $(p, r, \mathcal{F})$-Packing with $p \leq 2\lfloor r/2 \rfloor + 1$ admit almost linear kernels on every nowhere dense class of graphs, parameterized by the solution size k. As corollaries, we prove that Distance-r Vertex Cover, Distance-r Matching, $\mathcal{F}$-Free Vertex Deletion, and Induced-$\mathcal{F}$-Packing for any fixed finite family $\mathcal{F}$ of connected graphs admit almost linear kernels on every nowhere dense class of graphs. Our results extend the results for Distance-r Dominating Set by Drange et al. (STACS 2016) and Eickmeyer et al. (ICALP 2017) and for Distance-r Independent Set by Pilipczuk and Siebertz (EJC 2021).

Keywords: kernelization, independent set, dominating set, covering, packing