Splitting subspaces of Linear Operators over Finite Fields

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Let V be a vector space of dimension N over the finite field $\mathbb{F}_q$ and T be a linear operator on V. Given an integer m that divides N, an m-dimensional subspace W of V is T-splitting if $V = W \oplus TW \oplus \cdots \oplus T^{d-1}W$ where $d = N/m$. Let $\sigma(m, d; T)$ denote the number of m-dimensional T-splitting subspaces. Determining $\sigma(m, d; T)$ for an arbitrary operator T is an open problem. We prove that $\sigma(m, d; T)$ depends only on the similarity class type of T and give an explicit formula in the special case where T is cyclic and nilpotent. Denote by $\sigma_q(m, d; \tau)$ the number of m-dimensional splitting subspaces for a linear operator of similarity class type τ over an $\mathbb{F}_q$-vector space of dimension md. For fixed values of m, d and τ, we show that $\sigma_q(m, d; \tau)$ is a polynomial in q.

Keywords: splitting subspace, Krylov space, anti-invariant subspace, invariant subspace lattice, q-Vandermonde identity, finite field