Exact Enumeration of Self-Avoiding Walks on Non-vertex-transitive Lattices

Gaoran Yu* and John C. Wierman, Johns Hopkins University

A self-avoiding walk (SAW) is a walk that visits no vertex more than once. For any vertex-transitive graph G, its percolation threshold is bounded below by $\frac{1}{\sqrt[n]{c_n}}$, where c_n is the number of n-step SAWs starting from a fixed origin point in G. In 1957, Hammersley generalized the result to a large family of non-vertex-transitive graphs, which motivated us to bound their percolation threshold by counting the SAWs on them. There are no algorithms developed specifically for SAW enumeration on non-vertex-transitive graphs. Currently, the length-doubling method introduced by Schram, Barkema and Bisseling in 2011 is the most efficient algorithm for SAW enumeration on the cubic lattice. We generalize and parallelize the length-doubling method to non-vertex transitive lattices, including a type of face centered cubic (FCC) lattice and a type of body centered cubic (BCC) lattice. Our implementation is based on Map-Reduce programming in order to achieve a high parallel efficiency. This allows us to enumerate the SAWs up to length 12 on the FCC lattice and the BCC lattice by personal computer.