

Rigorous bounds relating bond percolation thresholds of two three-dimensional lattices

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A percolation model is an infinite random graph model for phase transitions and critical phenomena. The percolation threshold corresponds to a phase transition point, such as a melting or freezing temperature. The exact value of the percolation threshold is not known for any three-dimensional percolation models, which are important for physical applications. Furthermore, rigorous bounds for the percolation thresholds of three-dimensional models are quite poor. We relate the bond percolation models of two three dimensional lattices, the cubic lattice and a type of face-centered cubic lattice. We use the substitution method, which is based on stochastic ordering of probability measures on partition lattices, to translate bounds for one lattice into bounds for the other. In addition, the approach provides a lower bound for the difference between the bond percolation thresholds of the two lattices, without knowing very accurate bounds for either individual lattice.

Keywords: bond percolation, random graph, percolation threshold, stochastic ordering, partition lattice