Non-Monotonic Solutions to Nested Recursions: A Combinatorial Approach

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A *nested recursion* is any recursion where some argument contains a term of the recursion, for example, the Golomb (non-homogeneous) recursion $g(n) = g(n - g(n-1)) + 1$ with the initial condition $g(1) = 1$. A solution to a nested recursion with specified initial conditions is any sequence that satisfies the recursion together with the initial conditions; here $(g(n))_{n=1}^{\infty} = 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots$, the non-decreasing sequence of positive integers where each positive integer n occurs exactly n times.

In this talk we show how to derive a combinatorial interpretation for non-monotonic solutions to the generalized Golomb family $g_{s,j,\lambda}(n) = g_{s,j,\lambda}(n - s - g_{s,j,\lambda}(n-j)) + \lambda j$ with specified initial conditions. In so doing we extend a tree-based methodology that has been used exclusively to date to derive a combinatorial interpretation for the monotonic increasing solutions to several broadly defined families of nested recursions. We use this combinatorial interpretation to derive the asymptotic properties of the solutions.

This is ongoing joint work with Matthew Sunohara and Yvon Verberne.

Keywords: non-homogeneous nested recursion; tree-based solution methodology; generalized Golomb recursion.