Graph Convexity Parameters: Complexity and Applications

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We consider finite graphs. A *graph convexity* is a pair $(V(G), \mathcal{C})$, where $V(G)$ is the vertex set of a graph G, and $\mathcal{C}$ a family of subsets of $V(G)$, such that $\emptyset, V(G) \in \mathcal{C}$ and $\mathcal{C}$ is closed under intersections. The members of $\mathcal{C}$ are the *convex sets* of the graph. The main graph convexities are those based on paths of the graph, and named *path convexities*. Among the latter, the most common are the *geodetic, monophonic* and P_3 convexities, which correspond to those whose convex sets are closed under taking shortest paths, induced paths or common neighbors respectively, between any pair of vertices of a convex set. There are various graph convexity parameters, either derived from geometric and algebraic backgrounds, or within the context of computer science. Among the former, we can mention the classical Carathéodory number, Helly number, Radon number and rank. Besides, the latter are algorithmically inspired parameters, as the interval number, convexity number and hull number. In this talk, we discuss the complexity of determining all of such parameters, in the above convexities, mentioning both polynomial time and hardness results. In addition, we present some of the applications of the considered parameters.