Hamilton-Laceable Properties of Polygonal bigraphs

Gustavus J. Simmons
gsimmons30@comcast.net

The polygonal bigraphs, $P_{m,j}$, which include several well known families of graphs -- m-prisms, m-Mobius ladders, Weisstein's m-crossed prisms etc, are defined by the intersection of a centrally symmetric circle with the extended edges of a regular m-gon. There are $\lfloor (m-1)/2 \rfloor$ $P_{m,j}$ depending on the diameter of the defining circle. The Hamilton-laceable properties of the $P_{m,j}$ consolidate results for several of these families whose only common denominator had appeared to be that they were broadly concerned with extremal properties of Hamilton-laceable bigraphs; edge-minimal, edge-critical, edge-stable etc.

The secret to constructing Hamilton paths in $P_{m,j}$ is a mapping into hexagonal tilings on a cylinder. An easy way to visualize the mapping is to think of a tape consisting of m hexagons wound in a helix around a cylinder whose diameter is such that each wrap has j hexagons in it. This is possible for all values of $m > j$ and all $j > 1$. If $m \equiv 0 \bmod j$ the ends of the cylinder can be joined to form a torus. By construction, adjacencies of vertices in this hexagonal tiling of a torus are the same as in $P_{m,j}$; i,e., when $m \equiv 0 \bmod j$ $P_{m,j}$ is isomorphic to the cubic bigraph defined by a hexagonal tiling of a torus with wrapping numbers j and m/j.

The regularity of the mapping to a cylinder (torus) can be exploited to show $P_{m,j}$ is Hamilton laceable for many values of m and j. It is conjectured all $P_{m,j}$ are Hamilton laceable.