

Refined Turán numbers and Ramsey numbers for the loose 3-uniform path of length three

Andrzej Ruciński, Adam Mickiewicz University and Emory University

Let P denote a 3-uniform hypergraph on 7 vertices a, b, c, d, e, f, g and 3 edges $\{a, b, c\}$, $\{c, d, e\}$, and $\{e, f, g\}$. It has been conjectured that the Ramsey number $R(P; r) = r + 6$ for all $r \geq 1$. By a subtle analysis of the Turán numbers and extremal 3-graphs for P , we confirm that conjecture for $r \leq 9$. Along the way we introduce the notions of Turán numbers of ‘higher orders’ as well as the ‘conditional’ Turán numbers which might be of an independent interest. This is a joint work with E. Jackowska and J. Polcyn.

Keywords: 3-uniform hypergraph, loose path, Ramsey number, Turán number