Edge colorings of graphs without rainbow-subgraphs

Hanno Lefmann*, TU Chemnitz, Germany

Given r colors and a fixed complete graph F, an r-pattern P is a partition of the edge set of F into r (possibly empty) classes. An r-edge-coloring Δ of a (large) graph G is (F, P)-free, if G *does not* contain a subgraph isomorphic to F, where the partition of its edge set induced by Δ is isomorphic to P. The aim is to determine those n-vertex graphs (among all n-vertex graphs) that allow the largest number of (F, P)-free r-colorings.
Originally, this problem was first considered by Erdős and Rothschild in 1974, who asked whether edge-colorings avoiding a *monochromatic* copy of F lead to extremal configurations that are quite different from those of the Turán problem. For the compete graph $F = K_s$, $s \geq 3$, they conjectured that, for every n large enough, any n-vertex graph with the largest number of K_s-free 2-colorings is isomorphic to the Turán graph for K_s. This was proved for $s = 3$ by Yuster and later for any fixed $s \geq 4$ and $r \in \{2, 3\}$ by Alon, Balogh, Keevash, and Sudakov. For $r \geq 4$ colors the situation changes and extremal graphs are rarely known.
Here we study r-colorings that avoid *rainbow-colored* complete graphs $F = K_s$, where each edge of F is in a different class. In contrast to the monochromatic case, we were able, by using the regularity lemma, to find the optimal n-vertex graph for any large n and every fixed $r \geq r_0(F)$, which turns out to be the n-vertex Turán graph for F. This is joint work with Carlos Hoppen and Knuth Odermann.

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