On the structure of nonnegative matrices with a given spectrum

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Let $\sigma = (\lambda_1, \ldots, \lambda_n)$ be a list of complex numbers satisfying $|\lambda_1| > |\lambda_j|$ for $2 \leq j \leq n$ (Perron condition). Then there exists a nonnegative integer N such that, for all integers $m \geq N$, σ with m zeros added is the spectrum of a nonnegative nonderogatory $(n+m) \times (n+m)$ matrix A. We consider whether the same result holds with the Jordan form of A replaced by any Jordan form with the same characteristic polynomial.
Answering in the negative a question posed by Boyle and Handelman (1991), it was shown by Johnson, Laffey, and Loewy (2012) that in the case that σ is real, such an A cannot in general be chosen nonnegative and symmetric, and Laffey (1995) showed that A cannot in general be chosen nonnegative and diagonalizable. We show more generally that one cannot in general find such a nonnegative A with a given bound on the rank of the part of the Jordan form of A corresponding to the eigenvalue 0.
We also present some results obtained with Cronin showing that there are diagonalizable nonnegative matrices with nonzero real spectra which are not similar to nonnegative real symmetric matrices.

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