A problem in extremal set theory

Hans-Dietrich O.F. Gronau, University of Rostock, Germany

Let n be an integer and let N be an n-element set. We consider families $\mathcal{F}$ of subsets of N. We assume that any two sets of $\mathcal{F}$ have an non-empty intersection and their union is not the underlying set N. About 50 years ago several authors proved that $\max \mathcal{F} = 2^{n-2}$. About 40 years ago I was interested in families $\mathcal{F}$ in which any two sets satisfy the about condition or they are complementary. I was able to prove $\max \mathcal{F} = 2^{n-2}(1 + o(1))$. Moreover, I stated the conjecture

$$\max \mathcal{F} = \begin{cases} 2^{n-2} & if n \neq 2, 4, 6, 8 \\ \binom{n}{n/2} & if n = 2, 4, 6, 8 \end{cases}$$

This conjecture is still open. Recently, a student of mine settled a few small cases. The talk will present these results and will call the attention of the audience to this conjecture with the hope that someone will settle it completely soon.

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