

The Regular and Misère form of Edge Nim on Trees

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Edge Nim is a combinatorial game played on finite regular graphs with positive, integrally weighted edges. Two players alternately begin from a fixed starting vertex and move to an adjacent vertex, decreasing the weight of the incident edge to a strictly non-negative integer as they travel across it. The game ends when a player is confronted by a position where no incident edge has a nonzero weight (or, that is to say, when the player is unable to move). In the normal form, this player loses, and in the Misere form, this player wins. For the normal form of edge Nim on tree graphs, Fukuyama previously described a method using maximal matchings to determine the Grundy number for a given position. However, this did not produce a technique to articulate what the winning strategy might be. Clark then expanded on this, providing a method to answer those questions by using a pruning process. We present an alternate, algebraic approach to edge Nim on graphs for both the normal and misère forms, revealing not only a new method for attacking combinatorial games, but a complete, computationally inexpensive solution to Nim on tree graphs. This approach also suggests that the solution to Nim on trees may be a component of a larger algebraic structure resolving Nim on all graphs.

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