

On the Rank-unimodality of b -ary Partitions

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The b -ary partition is a naturally occurring combinatorial structure that represents a natural number n , given an integer b , as the sum of a series of powers of b . The set of all b -ary partitions of n forms a mathematical structure called a partially-ordered set, denoted as $R_b(n)$. Enumeration of these partitions and the characterization of $R_b(n)$ have been studied by many notable mathematicians, including Euler, Leibniz, and Knuth. $R_b(n)$ is ranked, meaning each b -ary partition of n is assigned an integer rank k , dependent on the number of powers of b in the partition. If the rank function of $R_b(n)$, which counts the number of partitions of rank k , peaks more than once, then n is considered non-unimodal in base b . We prove two lemmas, which we implement via a dynamic programming algorithm to enumerate this rank function and obtain data for large n . Our paper also proves the asymptotic greatest lower bound, b^4 , for the interval of non-unimodal n for all bases b . Additionally, we provide a closed-form expression for the difference between adjacent ranks of $R_b(n)$ for $n < b^3$. Our results characterize the topic of b -ary partitions, applicable to group theory, theoretical physics and computer science.