The Edge-Connectivity of the Square of a Graph

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Let G be a connected graph. The *edge-connectivity* $\lambda(G)$ of G is the minimum number of edges whose removal renders G disconnected. The *square* G^2 of G is the graph on the same vertex set as G, where two vertices are adjacent in G^2 if their distance in G is not more than two.
In this talk we present results on the edge-connectivity of the square of a connected graph. A classical result by Whitney states that $\lambda(G) \leq \delta(G)$, where $\delta(G)$ is the minimum degree of G. If $\lambda(G) = \delta(G)$ then G is said to be *maximally edge-connected*. Numerous sufficient conditions for a graph to be maximally edge-connected are known; the most famous one being due to Chartrand: if G has n vertices and satisfies $\delta(G) \geq \frac{1}{2}(n-1)$, then G is always maximally edge-connected.
In our talk we show that $\lambda(G^2) \geq \delta(G)+1$, and if G is bipartite we have the stronger bound $\lambda(G) \geq 2\delta(G)+1$. Among other results, we also show a version of Chartrand's conditions for maximally edge-connected squares: If $\delta(G) \geq \lfloor \frac{n+2}{4} \rfloor$ then G^2 is maximally edge-connected.

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