Isolation Number, Boolean Rank, and the Duality Gap

David Brown*, Utah State University

The **Boolean rank** of a $(0,1)$-matrix M is the minimum number of rank-1 matrices whose sum is M using Boolean arithmetic. The **isolation number** of a matrix is the maximum number of nonzero entries no two of which belong to the same line (row or column) and no two belong to a 2×2 submatrix all of whose entries are nonzero. The isolation number of a $(0,1)$-matrix is therefore a lower bound for its Boolean rank, but both are NP-complete as decision problems. When these parameters are formulated as linear programming problems, they are dual and therefore equal. But in their combinatorial formulation they are not necessarily equal, even in the case of very structured matrices such as tournament matrices. We investigate how wide the discrepancy between these parameters (isolation number and Boolean rank) can be, and uncover a relationship dictating when they are equal, via a certain derived graph being perfect, and can hence be solved using linear programming methods.

Keywords: Boolean rank, isolation number, fractional Boolean rank, perfect graph, biclique cover number, restricted matching.