Zero-reducible graphs and the König property

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In a recent paper Levit and Mandrescu gave a characterization of graphs having a unique perfect matching, which also satisfy the König property $\nu(G) = \tau(G)$. These graphs are exactly the ones that can be reduced to the empty graph by iteratively deleting a leaf vertex from the graph together with the vertex adjacent to it.
We enhance this reduction procedure by allowing the graph to be shrunk along an arbitrary subdividing vertex. The resulting delete-and-shrink reduction preserves the property of having a unique perfect matching, but not the König property. One would hope, however, for a simple connection between $\nu(G)$ and $\tau(G)$ in graphs that are reducible to the empty graph, so that $\tau(G)$ would be computable in polynomial time. We prove that this is not the case, and finding a minimum vertex cover is still NP-hard for such zero-reducible graphs. Nevertheless, we show that the zero-reducible property itself is decidable in linear time.

Keywords: graph reduction, König property, unique perfect matching, NP-completeness