Batch Codes from Hamming and Reed-Müller Codes

Felice Manganiello

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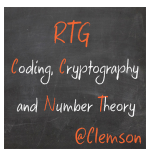
with T. Baumbaugh, Y. Diaz, S. Friesenhahn and A. Vetter.
Research financed by the NSF RTG grant (DMS #1547399).



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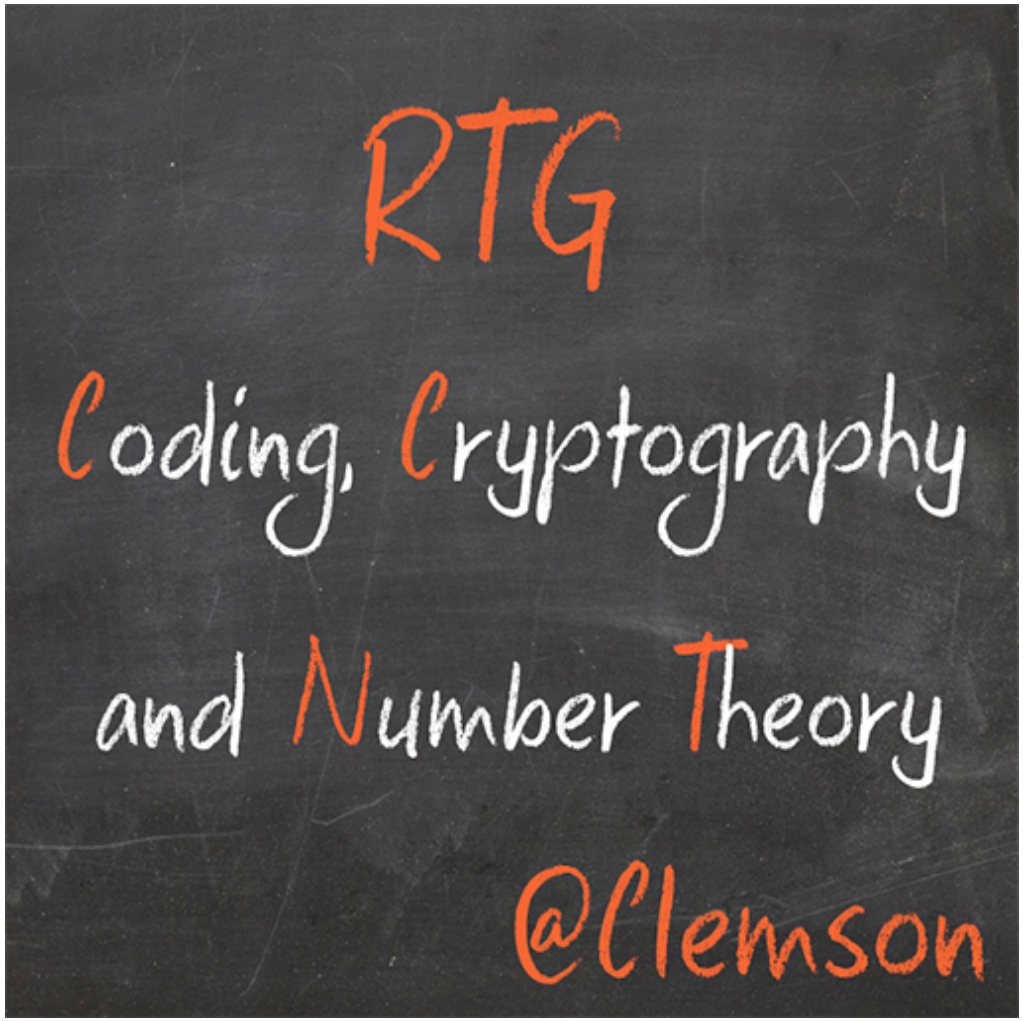
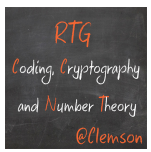
RTG: Coding Theory, Cryptography and Number Theory





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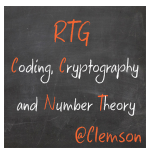

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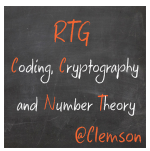
Previously on ...

Codes for Queueing Theory.

Suppose the new episode of Game of Thrones is available for downloads at time X. It is to be expected that at time X+epsilon an incredible amount of fan of the show would be requesting the episode. If the video is stored in only one server working on a first come first serve policy, then the waiting time for completing the download tasks becomes unbearable. How is it possible to decrease download times? This is a question of queueing theory, meaning the mathematical study of waiting lines, or queues. A better strategy is to store the episode in multiple servers. Some recent works show that lower download times are achievable perhaps using redundancy and coding theory.



Research direction: Impact of linear code in queuing theory.

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Coding Theory Application to Queueing and Download

PICTURE

EVENODD Codes [Blaum *et al.* '95]

Let $\mathcal{C}$ be a $[8, 4, 3]_q$ linear code with

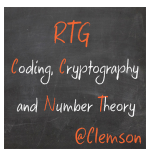
$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

EVENODD Codes [Blaum *et al.* '95]

Encode the word $m = (a_1, a_2, b_1, b_2) \in \mathbb{F}_q^4$ and obtain

$$mG = (a_1, a_2, b_1, b_2, a_1 + b_1, a_2 + b_2, a_2 + b_1, a_1 + a_2 + b_2) \in \mathbb{F}_q^8.$$

a_1	b_1	$a_1 + b_1$	$a_2 + b_1$
a_2	b_2	$a_2 + b_2$	$a_1 + a_2 + b_2$



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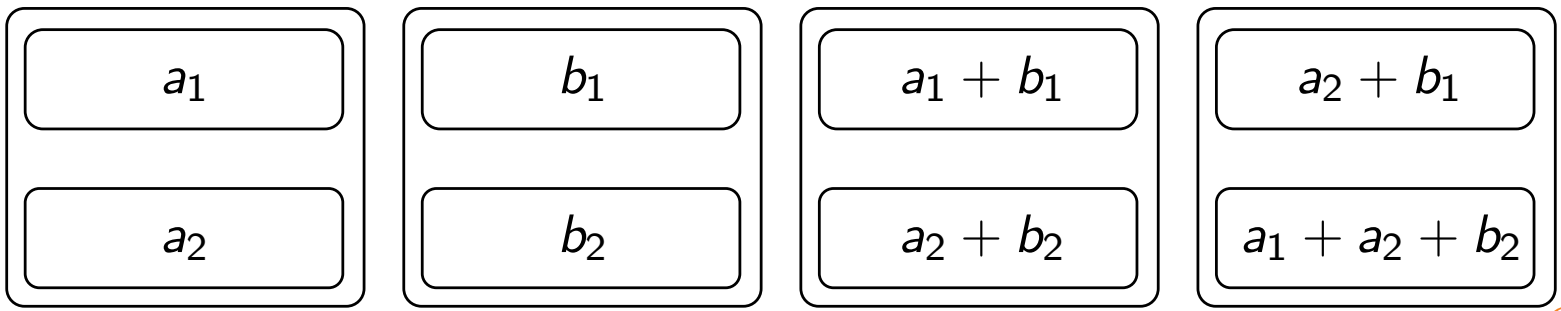
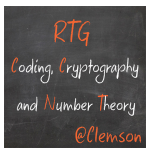
Store the information in different disks.



GOAL

- We have k chunks of data
- Encode as n chunks of data



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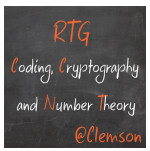

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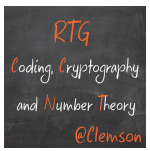
- There are t users downloading data
  - Could be distinct
  - Could allow repetitions
  - Could all be the same



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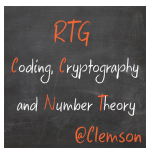

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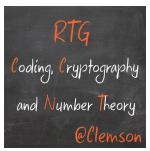
We want to be able to satisfy user demands regardless of what the data is



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Papers

- Y. Ishai, E. Kushilevitz, R. Ostrovsky, and A. Sahai. *Batch codes and their applications*. 2004
- M.B. Paterson, D.R. Stinson, and R. Wei. *Combinatorial Batch Codes*. 2008
- Helger Lipmaa, Vitaly Skachek. *Linear Batch Codes*. 2015
- Hui Zhang and Vitaly Skachek. *Bounds for Batch Codes with Restricted Query Size*. 2016
- N. Silberstein and A. Gál. *Optimal combinatorial batch codes based on block designs*. 2016
- Eldho K. Thomas and Vitaly Skachek. *Constructions and Bounds for Batch Codes with Small Parameters*. 2017
- Hui Zhang, Eitan Yaakobi, and Natalia Silberstein. *Multiset combinatorial batch codes*. 2018

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Batch Codes [Ishai *et al.* '04]

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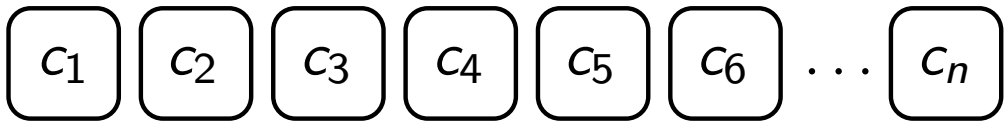

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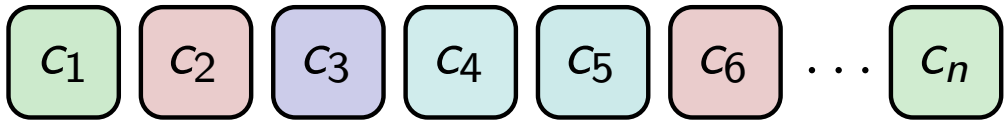

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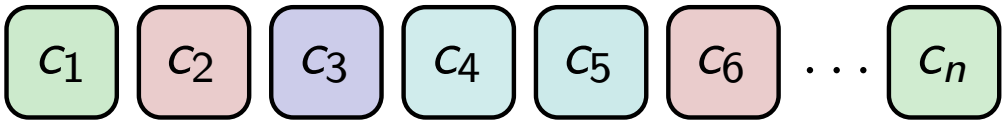

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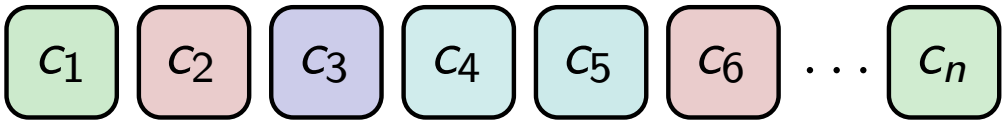

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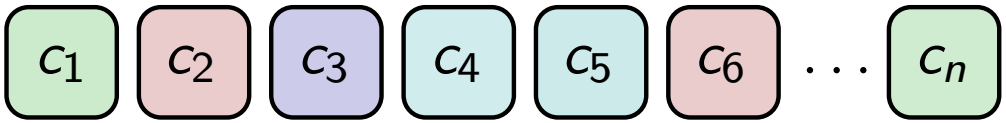

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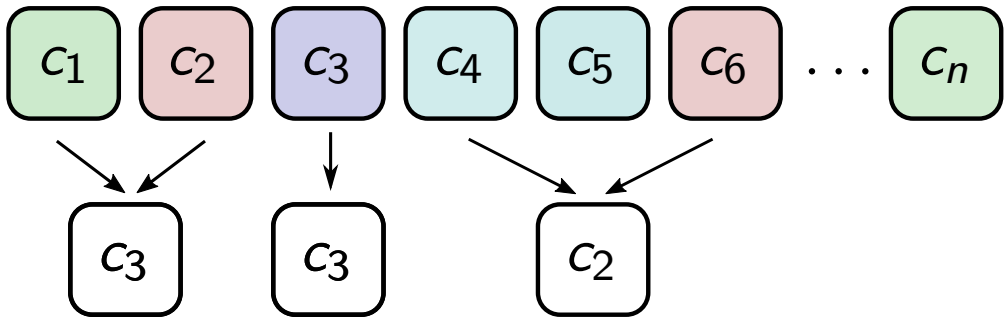
  - $\left|(\bigcup_{s=1}^{t} A_{i_s}) \cap B_\ell\right| \leq \tau$ for $\ell \in [m]$



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Simple Example

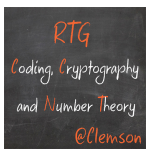
Consider a $[5, 3]$ code over $\mathbb{F}_2$ with parity check matrix

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

and buckets

$$\{1, 2\}, \{3, 4\}, \{5\}.$$

- This gives us the equations:
  - $c_1 + c_3 + c_5 = 0$
  - $c_2 + c_4 + c_5 = 0$
- If we want to recover $(1, 2)$, we can take
  - $A_1 = \{1\}$
  - $A_2 = \{4, 5\}$



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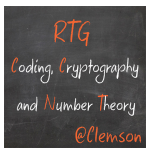

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A bound on the download bandwidth

Theorem (Bandwidth bound)

Let $\mathcal{C}$ be an $[n, k, t, m, \tau]$ linear batch code with minimal locality r. Then it holds that

$$m\tau \geq (t-1)r + 1.$$

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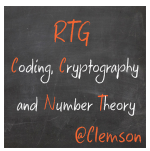
- Try to recover t copies of it
  - One direct download
  - The remaining $t-1$ copies require at least r downloads



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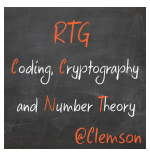

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Definition

A n $[n, k, t, m, \tau]$ linear batch code $\mathcal{C}$ with minimal locality r is optimal if it satisfies the Bandwidth bound with equality.

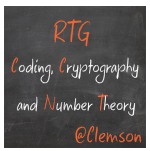
Locality based on the dual code

Lemma

Let $\mathcal{C} \subseteq \mathbb{F}_q^n$ be a linear code and $d^\perp$ be the minimum distance of $\mathcal{C}^\perp$. If $\mathcal{C}^\perp$ is generated by its minimum weight codewords and

$$\bigcup_{\lambda \in \mathcal{C}^\perp} \text{supp}(\lambda) = [n],$$

then $\mathcal{C}$ is a locally recoverable code with locality $d^\perp - 1$.



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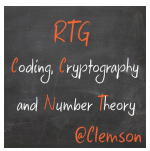

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Definition

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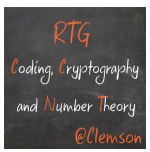
Let $s \geq 2$, $n = 2^s - 1$ and $H \in \mathbb{F}_2^{2^s-1 \times s}$ be a matrix whose columns are all of the nonzero vectors of $\mathbb{F}_2^s$. The binary Hamming code is

$$\mathcal{H}_s = \ker(H^t).$$

$\mathcal{H}_s$ is a $[2^s - 1, 2^s - 1 - s, 3]$ linear code.

If $s = 3$, then the parity check matrix H is as follows:

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$



Then $\mathcal{H}_s$ is a $[7, 4, 3]$ linear code.

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Locality and Availability

Lemma

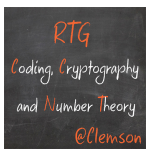
The Hamming code $\mathcal{H}_s$ has locality $2^{s-1}-1$ and availability 1.

It is well known that $\mathcal{H}_s^{\perp}$ is a $[2^s-1, s, 2^{s-1}]$ linear code that satisfies the *Dual minimum codewords condition.* then $\mathcal{H}_s$ has locality $2^{s-1}-1$.

Suppose $\mu = 3$ wlog for $i = 1$, then there are two minimum weight codewords $\lambda_1, \lambda_2 \in \mathcal{C}^{\perp}$ such that

$$\text{supp}(\lambda_1) \cap \text{supp}(\lambda_2) = \{1\}$$

Since $\mathcal{C}^{\perp}$ is linear and $\mathbf{1} \in \mathcal{C}^{\perp}$, then $1 + \lambda_1 + \lambda_2 \in \mathcal{C}^{\perp}$.



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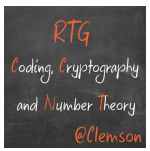
Batch Properties

Theorem

A binary $[n, k]$ Hamming code is an optimal batch code with properties $[n, k, 2, m, \tau]$, for any $m, \tau \in \mathbb{N}$ such that $m\tau = 2^{s-1}$.



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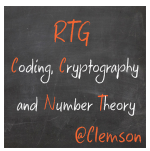
Let $h_j \in \mathbb{F}_2^s \setminus \{0\}$ for $1 \leq j \leq n$.

- If $h_a + h_b = \mathbf{1}$, $a\&b$ same bucket.
- Since $h_n = \mathbf{1}$ in H, n own bucket.



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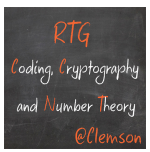
Buckets:

$\{1, 6\}, \{2, 5\}, \{3, 4\}, \{7\}$



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Batch Properties

Theorem

A binary $[n, k]$ Hamming code is an optimal batch code with properties $[n, k, 2, m, \tau]$, for any $m, \tau \in \mathbb{N}$ such that $m\tau = 2^{s-1}$.

Buckets construction.

Let $h_j \in \mathbb{F}_2^s \setminus \{0\}$ for $1 \leq j \leq n$.

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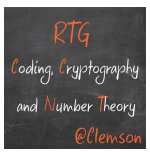
Buckets:

$\{1, 6\}, \{2, 5\}, \{3, 4\}, \{7\}$

The support of any $\lambda \in \mathcal{C}^{\perp}$ intersects each bucket in a singleton.



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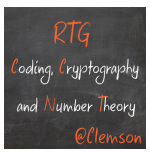

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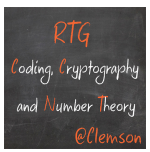

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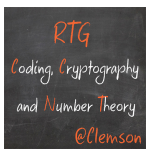

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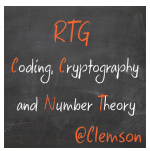

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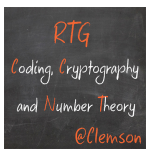
Definition

Definition

Let $\mathbb{F}_2[x_1, \ldots, x_\mu]$, $\mathbb{F}_2^\mu = \{P_1, \ldots, P_n\}$ and $\rho < \mu$.
The binary Reed-Müller code, $\mathcal{RM}(\rho, \mu)$ is defined as:

$$\mathcal{RM}(\rho, \mu) = \{(f(P_1), \ldots, f(P_n)) \mid f \in \mathbb{F}_2[X_1, \ldots, X_\mu]^\rho\},$$

where $\mathbb{F}_2[x_1, \ldots, x_\mu]^\rho$ is the set of polynomials of total degree at most ρ.
$\mathcal{RM}(\rho, \mu)$ is a $[n = 2^\mu, k = \sum_{i=0}^{\rho} \binom{\mu}{i}]$ linear code.



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Alternative definition

Definition

Let $\rho < \mu$ and $n = 2^\mu$. A binary Reed-Müller code $\mathcal{RM}(\rho, \mu)$ is

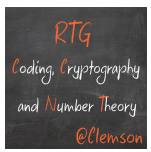
$$\mathcal{RM}(\rho, \mu) = \{(u \mid u+v) \mid u \in \mathcal{RM}(\rho, \mu-1),\ v \in \mathcal{RM}(\rho-1, \mu-1)\}$$

where $\mathcal{RM}(0, \mu) = \{0, \mathbf{1}\} \subset \mathbb{F}_2^n$, and $\mathcal{RM}(\mu, \mu) = \mathbb{F}_2^n$.

As a consequence if $G_{\rho,\mu}$ is a generator matrix of the code $\mathcal{RM}(\rho, \mu)$, then

$$G_{\rho,\mu} = \begin{pmatrix} G_{\rho,\mu-1} & G_{\rho,\mu-1} \\ 0 & G_{\rho-1,\mu-1} \end{pmatrix}$$

where $G_{0,\mu} = \mathbf{1}$ and $G_{\mu,\mu} = I$.



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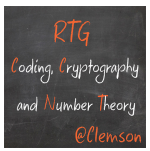
Properties of Reed-Müller codes

Theorem

The dual of a Reed-Müller code $\mathcal{RM}(\rho, \mu)$ is the Reed-Müller code $\mathcal{RM}(\mu - 1 - \rho, \mu)$.

Corollary (Case of Thm by Ding-Key 2000)

Binary Reed-Müller codes are generated by their minimal weight codewords.



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Properties of $\mathcal{RM}(1,\mu)$

Theorem

Let $\mathbb{F}_q^\mu = \{P_1, \ldots, P_n\}$ be the set of evaluation points for $\mathcal{RM}(1,\mu)$. Then

$$(\lambda_1, \ldots, \lambda_n) \in \mathcal{RM}(1,\mu)^\perp \iff \sum_{i=1}^{n} \lambda_i P_i = 0 \ \textit{and} \ \sum_{i=1}^{n} \lambda_i = 0.$$

Corollary

The minimum distance of $\mathcal{RM}(1,\mu)^\perp$ is 4.

If $P_1 = 0$, $P_i, P_j \in \mathbb{F}_2^\mu$ and $P_\ell = P_i + P_j$ then

$$P_1 + P_i + P_j + P_\ell = 0.$$

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Locality and Availability

Theorem

$\mathcal{RM}(1,\mu)$ is a locally repairable code with locality 3 with availability

- $\frac{2^\mu-1}{3}$ *when* μ *is even, and*
- *at least* $\frac{2^\mu-4}{4}$ *when* μ *is odd.*

Batch properties of $\mathcal{RM}(1, \mu)$

Theorem

Any first order Reed-Müller code, $\mathcal{RM}(1, \mu)$, with $\mu \geq 4$, has batch properties $(n, k, 4, m, \tau)$ for any $m, \tau \in \mathbb{N}$ such that $m\tau = 10$.

Any $\mathcal{RM}(1, \mu)$ is an optimal batch code when $t = 4$.

Theorem

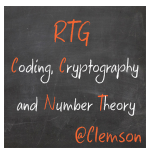
Let $\rho \geq 1$ and $\mu \in \{2\rho + 2, 2\rho + 3\}$. Then, for $\rho' \leq \rho$, $\mathcal{RM}(\rho', \mu)$ is a $(n, k, 4, m, \tau)$ linear batch code for any $m, \tau \in \mathbb{N}$ such that $m\tau = 10 \cdot 2^{2\rho - 2}$.

Conclusions and projects

- Batch codes can be considered as a generalization of codes with locality and availability.
- Hamming codes are optimal batch codes for 2-strings download.
- First order Reed-Müller codes are optimal batch codes for 4-strings download.

Projects

- Find constructions of batch codes for any request from t users.
- Influence of codes with high availability on the McEliece Cryptosystem.



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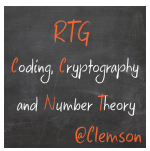

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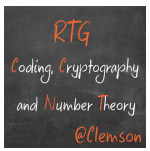

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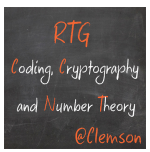

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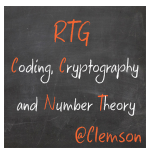

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Muller vs. Müller

Portrait of David E. Muller, professor of mathematics and computer science at the University of Illinois from 1953-92.

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