MATH DAY 2006 at FAU

Competition A–Individual

1. The degree of $(x^2+5)^5(x^3+x+1)^3$ as a polynomial in x is

   (A) 19 **(B)** 90 **(C)** 13 **(D)** 42 **(E)** NA

2. The coefficient of x^5 in the polynomial expansion of $(x^5+2x^4+3x^3+4x^2+3x+2)^2$ is

   (A) 1 **(B)** 15 **(C)** 25 **(D)** 30 **(E)** NA

3. The polynomial x^4-5x^2+x is divided by the polynomial x^2+4. The remainder is

   (A) 0 **(B)** $x+36$ **(C)** $x-36$ **(D)** $x+9$ **(E)** NA

4. Suppose the equation $x^2+ax+b=0$ has two distinct solutions x_1, x_2. Then $x_1^2+x_2^2$ equals:

   (A) a^2 **(B)** a^2-2b **(C)** a^2+2b **(D)** $-a^2$ **(E)** NA

5. If $x-2$ divides x^8-ax+2, then a is

   (A) 255 **(B)** 127 **(C)** 129 **(D)** 343 **(E)** NA

6. Let x be a real number such that

   $$\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}=3.$$

   Determine the value of $x+\dfrac{1}{x}$.

   (A) 18 **(B)** $9+2\sqrt{5}$ **(C)** $9+3\sqrt{5}$ **(D)** $18-\sqrt{5}$ **(E)** NA

7. Suppose $f(x)=x^3+3x^2-2$. Compute $f(f(0))$.

   (A) 2 **(B)** -2 **(C)** 0 **(D)** 1 **(E)** NA

8. If $f(x+y)=f(x)+f(y)$ and $f(4)=3$, then $f(1)$ is

   (A) 1 **(B)** 0 **(C)** $\dfrac{4}{3}$ **(D)** $\dfrac{3}{4}$ **(E)** NA

9. Determine the last digit of $3^{2006}+7^{2006}$.

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** NA

10. The integers

$$1, 3, 4, 9, 10, 12, 13, 27, 28, 30, \ldots$$

are the sums of distinct powers of 3 arranged in increasing order. What is the 100th term of the sequence?

(A) 100 **(B)** 345 **(C)** 543 **(D)** 981 **(E)** 1024 **(F)** NA

11. Suppose $a > 0$ and $(a + bi)^2 = 5 + 12i$. Then a equals

(A) 2 **(B)** 3 **(C)** $\sqrt{5}$ **(D)** $\sqrt{3}$ **(E)** NA

12. Determine the value of

$$\left(\sqrt{3} + i\right)^{2006}$$

(A) 2^{2006} **(B)** $2^{2005}\left(\sqrt{3} + i\right)$ **(C)** $2^{2005}\left(1 + i\sqrt{3}\right)$ **(D)** 0 **(E)** NA

13. Suppose that a, b, c are real numbers, $a > 1, b > 1$, $c > 0$. Suppose that $\log_a b = 0.6$ and $\log_b c = 3$. Then $\log_a c$ equals

(A) 0.18 **(B)** 3.6 **(C)** 0.5 **(D)** 0.2 **(E)** NA

14. The equation

$$2\log_7(x + 3) - \log_7(x + \frac{3}{2}) = 1$$

(where $\log_7$ denotes the log in base 7) has two real solutionsx_1, x_2. Which of the following numbers is **closest** to $|x_1 - x_2|$?

(A) 2.0 **(B)** 2.2 **(C)** 2.4 **(D)** 2.6 **(E)** 2.8

15. The number 715 can be written as a sum of k consecutive numbers. Which of the following can**not** be k?

(A) 5 **(B)** 7 **(C)** 10 **(D)** 11 **(E)** 13 **(F)** NA

16. If $5\cos\theta + 12\sin\theta = 13$, then $\tan\theta =$

(A) -1 **(B)** 1 **(C)** $\frac{5}{12}$ **(D)** $\frac{12}{5}$ **(E)** NA

17. Let α, β be the acute angles of a right triangle. Then $(\tan\alpha)(\tan\beta)$ equals

(A) 1 **(B)** -1 **(C)** 0 **(D)** $\sqrt{2}$ **(E)** NA

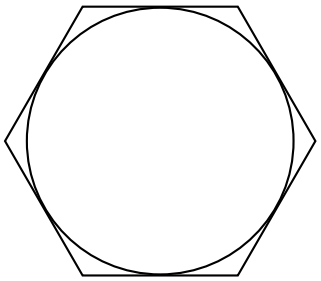
18. A rectangle has dimensions 20 inches and 40 inches. The width is increased by 20 percent and the breadth is decreased by 20 percent. What is the percentage change in the area?

(A) an increase of 4 percent **(B)** a decrease of 4 percent **(C)** an increase of 20 percent
(D) a decrease of 20 percent **(E)** NA

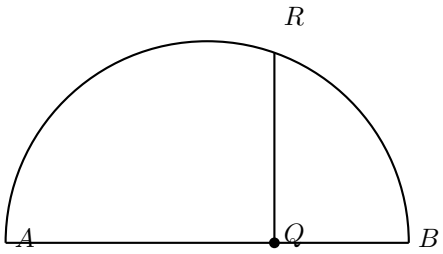
19. A circle goes through the points of coordinates $(3, -6)$, $(4, -1)$ and $(3, 0)$. Determine its radius.

(A) 6 **(B)** $\sqrt{6}$ **(C)** 3 **(D)** $\sqrt{13}$ **(E)** NA

20. Determine the perimeter of a regular hexagon circumscribed about a circle of radius 1.

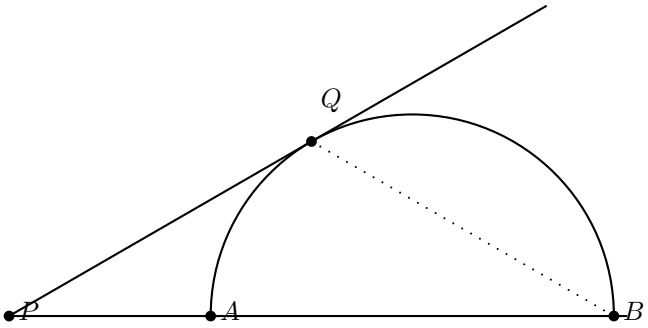


21. A point Q is marked off on the segment AB and a perpendicular to AB is drawn at Q intersecting the semicircle of base AB at R. If the lengths of AB and AQ are $|AB| = 13$, $|AQ| = 9$, determine the length $|QR|$ of QR.



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** NA

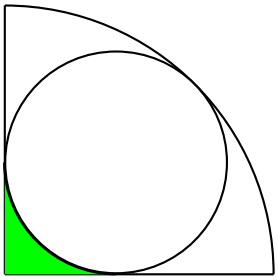
22. The point P lies on the same line as the base AB of a semicircle of radius r. The tangent line to this semicircle through P touches the semicircle at Q.



If $r = 7$ and the length of PA is 11, determine the length of the segment QB.

(A) $\frac{21}{2}$ **(B)** $\frac{28}{3}$ **(C)** $\frac{35}{3}$ **(D)** $\frac{42}{5}$ **(E)** NA

23. A circle is inscribed in a quadrant of a circle of radius 4. Find the area of the shaded region.

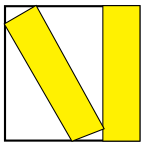


(A) $4(7-2\pi)(5-2\sqrt{2})$ **(B)** $4(4-\pi)(3-\sqrt{2})$ **(C)** $4-\pi$ **(D)** $4(4-\pi)$ **(E)** NA

24. A triangle has sides of lengths 13, 14, and 15, respectively. Determine its area.

(A) 80 **(B)** 84 **(C)** 91 **(D)** 105 **(E)** NA

25. On a square book shelf there are two identical books arranged as in the diagram below. If each book has height 1 unit, what is its thickness?



(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $2-\sqrt{3}$ **(D)** $3-\sqrt{2}$ **(E)** NA