Implementing Joux-Vitse's Crossbred Algorithm for Solving MQ Systems over $\mathbb{F}_2$ on GPUs

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  - Scheme-specific

Goal

Generic attack on MQ over $\mathbb{F}_2$

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Generic attack on MQ over $\mathbb{F}_2$

- Schemes built on MQ over $\mathbb{F}_2$: UOV, MQ-hash, MQ-PKI
- Substitute variables and introducing equations: MP $\rightarrow$ MQ
- With Weil descent: MQ over $\mathbb{F}_{2^n} \rightarrow$ MQ over $\mathbb{F}_2$

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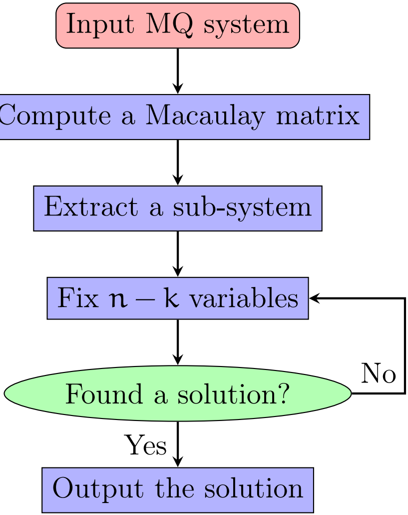
The Original Crossbred Algorithm



Crossbred - Macaulay Matrix

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  - Each row in a Macaulay matrix is the product of a polynomial f in $\mathcal{F}$ by a monomial t such that $\deg(t \cdot f) \leqslant D$.

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- Example

$$\mathcal{F} = \begin{cases} f_1 = x_1x_2 + x_2x_3 + x_3 \\ f_2 = x_1x_4 + 1 \end{cases}$$

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Crossbred - Macaulay Matrix

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	$x_1x_2x_3x_4$	$x_1x_2x_3$	$x_1x_2x_4$	$x_1x_3x_4$	$x_2x_3x_4$	x_1x_2	x_1x_3	x_2x_3	x_1x_4	x_2x_4	x_3x_4	x_1	x_2	x_3	x_4	1
$x_1x_2 \cdot f_1$						1										
$x_1x_2 \cdot f_2$			1			1										
$x_1x_3 \cdot f_1$							1									
$x_1x_3 \cdot f_2$				1			1									
$x_2x_3 \cdot f_1$		1														
$x_2x_3 \cdot f_2$	1							1								
$x_1x_4 \cdot f_1$	1		1	1												
$x_1x_4 \cdot f_2$																
$x_2x_4 \cdot f_1$			1													
$x_2x_4 \cdot f_2$			1							1						
$x_3x_4 \cdot f_1$	1				1						1					
$x_3x_4 \cdot f_2$				1							1					
$x_1 \cdot f_1$		1				1	1									
$x_1 \cdot f_2$									1			1				
$x_2 \cdot f_1$						1										
$x_2 \cdot f_2$			1										1			
$x_3 \cdot f_1$		1						1						1		
$x_3 \cdot f_2$				1										1		
$x_4 \cdot f_1$			1		1						1					
$x_4 \cdot f_2$									1						1	
$1 \cdot f_1$						1		1						1		
$1 \cdot f_2$									1							1

-

Crossbred - Extracted Sub-system

- Consists of only monomials that become linear in $x_1, \ldots x_k$ after fixing $x_{k+1}, \ldots, x_n$.

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- Example: Fixing $(x_3, x_4) = (0, 0)$ in

$$\mathcal{S} = \begin{cases} x_1x_4 + x_2x_3 + x_1 + x_3 + x_4 = 0 \\ x_1x_3 + x_3x_4 + x_2 + 1 = 0 \\ x_2x_3 + x_2x_4 + x_3x_4 + x_1 + x_4 = 0 \end{cases}$$

yields

$$\mathcal{S}' = \begin{cases} x_1 = 0 \\ x_2 + 1 = 0 \\ x_1 = 0 \end{cases}$$

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- Basic idea:
  $x_1x_2 + x_1x_3 + x_2x_3 + x_3 + 1 = (x_1x_2 + 1) + x_3(x_1 + x_2 + 1)$

Crossbred - Fast Evaluate

- Basic idea:
  $x_1x_2 + x_1x_3 + x_2x_3 + x_3 + 1 = (x_1x_2 + 1) + x_3(x_1 + x_2 + 1)$
- Apply this technique recursively to fix $n - k$ variables

- How to extract a sub-system?

- How to extract a sub-system?
- How to check the solvability of a linear system?

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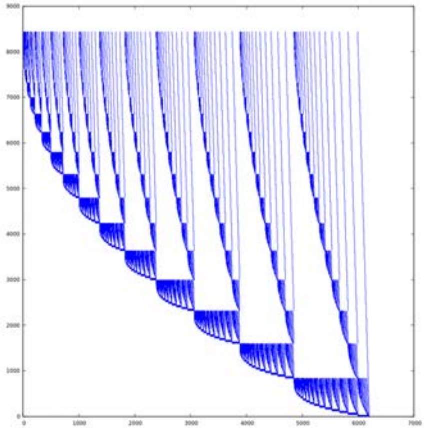
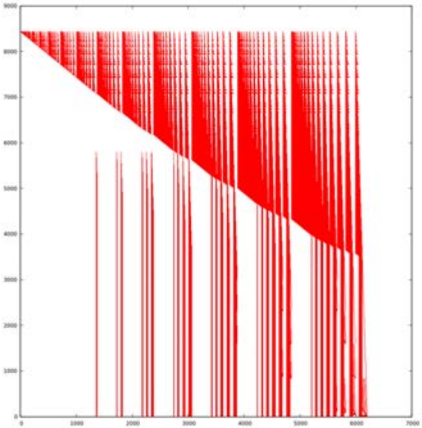
Gaussian elimination?



(a) Original Macaulay Matrix



(b) After Gaussian Elimination

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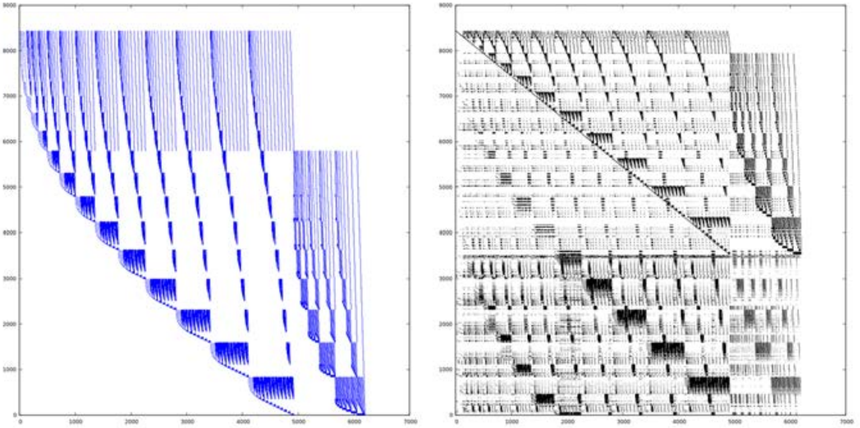
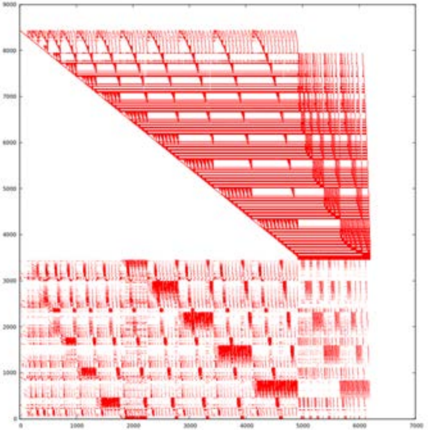
Permute columns and swap rows



(a) Permuted Macaulay Matrix

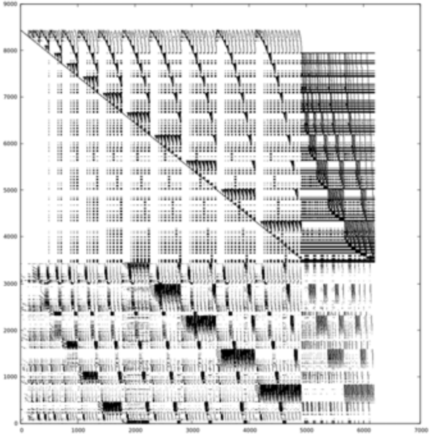
(b) After Row-swapping

Ignore the lower part



(c) After Gaussian Elimination

Reduce dimension



(d) After Eliminating Pivot Monomials

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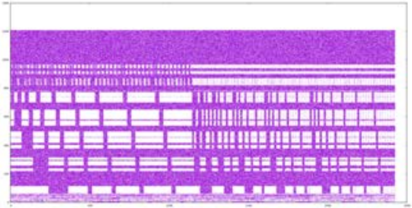
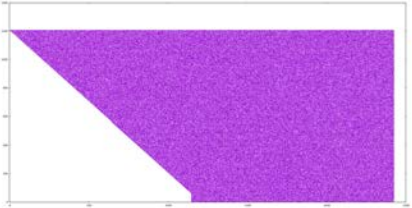
- Reduced Macaulay matrix



(a) Initial Reduced Macaulay Matrix



(b) After Gaussian Elimination

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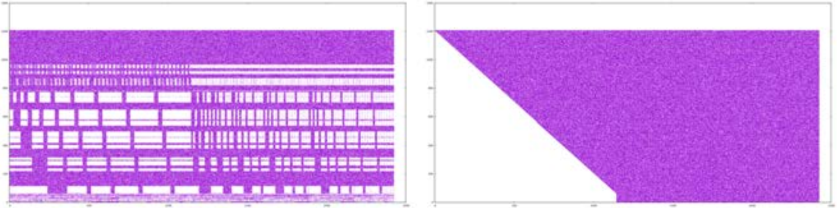
Adaption - Extracting a Sub-system

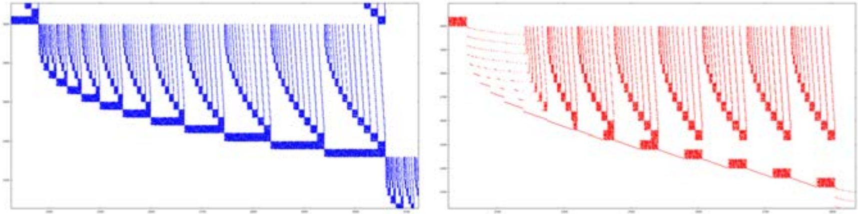
- Reduced Macaulay matrix



(a) Initial Reduced Macaulay Matrix

(b) After Gaussian Elimination

- A rule of thumb: Gauss-Jordan elimination



(a) Without Gauss-Jordan Elimination

(b) With Gauss-Jordan Elimination

- Observation:

$$f(\vec{a}) = f(\vec{a'}) + \frac{\partial f}{\partial x_i}(\vec{a'}).$$

when $\vec{a}$ and $\vec{a'}$ differs only by one coordinate.

Adaption - Fixing Variables in the Sub-system

- Observation:

$$f(\vec{a}) = f(\vec{a'}) + \frac{\partial f}{\partial x_i}(\vec{a'}).$$

  when $\vec{a}$ and $\vec{a'}$ differs only by one coordinate.
- For sub-system $\mathcal{S}$ of m degree-D equations in n variables, fix $n - k$ variables in $\mathcal{S}$ takes $\mathcal{O}(D \cdot k)$ instructions.

Example

$$f(x_4, x_5, x_6, x_7) = x_1x_4x_5x_6 + x_1x_4x_5x_7 + x_4x_5x_6x_7 + x_1x_4x_5 + x_2x_4x_6 + x_4x_6x_7 + x_1x_4 + x_1x_5 + x_5x_7 + x_6x_7 + x_1 + x_2 + x_4 + 1$$

$$\frac{\partial f}{\partial x_4} = x_1x_5x_6 + x_1x_5x_7 + x_5x_6x_7 + x_1x_5 + x_2x_6 + x_6x_7 + x_1 + 1$$

$$\frac{\partial^2 f}{\partial x_4 \partial x_7} = x_1x_5 + x_5x_6 + x_6$$

$$\frac{\partial^3 f}{\partial x_4 \partial x_6 \partial x_7} = x_5 + 1$$

Adaption - Fixing Variables in the Sub-system

$$
\begin{aligned}
f(0,0,0,0) &= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,0) \\
&= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,1) + \frac{\partial^2 f}{\partial x_4 \partial x_7}(1,0,0,1) \\
&= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,1) + \frac{\partial^2 f}{\partial x_4 \partial x_7}(1,0,1,1) + \frac{\partial^3 f}{\partial x_4 \partial x_6 \partial x_7}(1,0,1,1) \\
&= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,1) + \frac{\partial^2 f}{\partial x_4 \partial x_7}(1,0,1,1) + \frac{\partial^3 f}{\partial x_4 \partial x_6 \partial x_7}(1,0,1,0) + \frac{\partial^4 f}{\partial x_4 \partial x_6 \partial x_7 \partial x_7} \\
&= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,1) + \frac{\partial^2 f}{\partial x_4 \partial x_7}(1,0,1,1) + \frac{\partial^3 f}{\partial x_4 \partial x_6 \partial x_7}(1,0,1,0) + 0 \\
&= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,1) + \frac{\partial^2 f}{\partial x_4 \partial x_7}(1,0,1,1) + 1 \\
&= f(1,0,0,0) + \frac{\partial f}{\partial x_4}(1,0,0,1) + 1 + 1 \\
&= f(1,0,0,0) + x_1 + 1 + 1 + 1 \\
&= x_2 + x_1 + 1 + 1 + 1 \\
&= x_1 + x_2 + 1
\end{aligned}
$$

because $1010 \to 1011 \to 1001 \to 1000 \to 0000$

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Adaption - Testing a Linear System

Gauss-Jordan elimination?

$$\mathcal{S} = \begin{array}{c} \\ eq_1 \\ eq_2 \\ eq_3 \\ eq_4 \\ eq_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & 1 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \end{array} \xrightarrow[\text{no swapping}]{1^{\text{st}} \text{ pivot element}} \begin{array}{c} eq_1 \\ eq_2 \\ eq_3 \\ eq_4 \\ eq_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \xrightarrow[eq_3,\ eq_4]{\text{reduce with}} \begin{array}{c} eq_1 \\ eq_2 \\ eq_3 \\ eq_4 \\ eq_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow[\text{swap } eq_2 \text{ and } eq_3]{2^{\text{nd}} \text{ pivot element}} \begin{array}{c} eq_1 \\ eq_3 \\ eq_2 \\ eq_4 \\ eq_5 \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \xrightarrow[eq_1,\ eq_4,\ eq_5]{\text{reduce with}} \begin{array}{c} eq_1 \\ eq_3 \\ eq_2 \\ eq_4 \\ eq_5 \end{array} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow[\text{swap } eq_2 \text{ and } eq_4]{3^{\text{rd}} \text{ pivot element}} \begin{array}{c} eq_1 \\ eq_3 \\ eq_4 \\ eq_2 \\ eq_5 \end{array} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \xrightarrow[eq_3,\ eq_5]{\text{reduce with}} \begin{array}{c} eq_1 \\ eq_3 \\ eq_4 \\ eq_2 \\ eq_5 \end{array} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow[\text{no swapping}]{4^{\text{th}} \text{ pivot element}} \begin{array}{c} eq_1 \\ eq_3 \\ eq_4 \\ eq_2 \\ eq_5 \end{array} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow[eq_1,\ eq_5]{\text{reduce with}} \begin{array}{c} eq_1 \\ eq_3 \\ eq_4 \\ eq_2 \\ eq_5 \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A better variant:

- $\mathcal{O}(k^2)$ instructions for testing
- $\mathcal{O}(k)$ instructions for extracting a solution

$$
\begin{array}{c} \text{mask} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}
\quad
\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}
\xrightarrow[\text{reduce with } eq_3,\ eq_4]{1^{\text{st}} \text{ pivot row: } eq_1}
\begin{array}{c} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}
\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}
\xrightarrow[\text{reduce with } eq_1,\ eq_4,\ eq_5]{2^{\text{nd}} \text{ pivot row: } eq_3}
\begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{array}
\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}
$$

$$
\xrightarrow[\text{reduce with } eq_3,\ eq_5]{3^{\text{rd}} \text{ pivot row: } eq_4}
\begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{array}
\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}
\xrightarrow[\text{reduce with } eq_1,\ eq_5]{4^{\text{th}} \text{ pivot row: } eq_2}
\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array}
\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

- Split search space into N smaller sub-spaces and launch N threads?

Adaption - Parallelization

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  - Same enumeration starting point
  - Same last order partial derivatives

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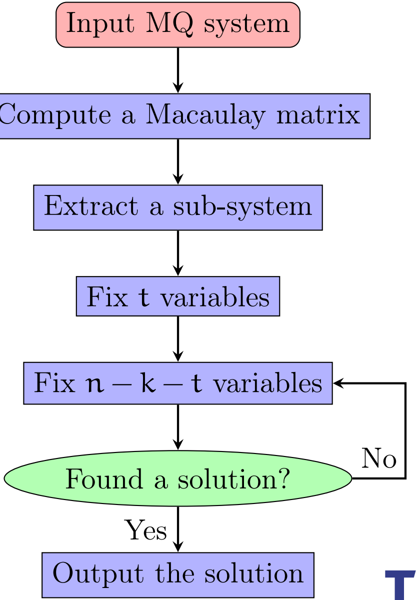
The Adapted Crossbred Algorithm



- From scratch

MQsolver

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- Part on CPU, part on GPU $\rightarrow$ pipeline

Environment

The Saber cluster:

https://blog.cr.yp.to/20140602-saber.html

- GTX 780 / GTX 980
- AMD FX-8350 @4GHz

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Fukuoka MQ type 1 challenges: $n = 55$ to 74, $m = 2n$

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$m = 2n$

For $n = 67$:

- Crossbred: 6200 CPU-hours (Xeon 2690)

For $n = 74$:

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- Parallel Crossbred: 98.39 GPU-hours (GTX 980)

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- Parallel Crossbred: 8236.05 GPU-hours

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- MQ system where $n = 84$, $m = 80$
- GPU: Nvidia GTX 980
- At most 37000, on average: 3600 (GPU-years)
- Claimed to have 80 bits of security, in fact only 76.5 bits

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- Parallel Crossbred outperforms the orginal Crossbred algorithm by a factor as high as ≈ 60.

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- We solved all the Fukuoka MQ type I challenges, $n = 55$ to 74.
- We showed that a post-quantum cryptographic scheme whose security relies on an MQ system where $n = 84$ and $m = 80$ can be broken with 37000 GPUs in at most one year, and on average 35 days.

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- Parallel Crossbred outperforms the orginal Crossbred algorithm by a factor as high as ≈ 60.
- We solved all the Fukuoka MQ type I challenges, $n = 55$ to 74.
- We showed that a post-quantum cryptographic scheme whose security relies on an MQ system where $n = 84$ and $m = 80$ can be broken with 37000 GPUs in at most one year, and on average 35 days.
- We showed that an MQ system with only 80 bits of security should not be considered secure anymore.

Thank you for attention!

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