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On sharp higher order Sobolev embeddings. (English. English summary)

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The authors prove a symmetrization principle which helps to overcome the lack of a Pólya-Szegő principle for higher order derivatives. Namely, they show that for many rearrangement invariant (r.i.) function spaces Y the inequality $\|t^{-k/n}(f^{**}(t)-f^{*}(t))\|_Y \le C\|\nabla^k f\|_Y$ holds for each $f \in W_0^{k,Y}(\Omega)$ (Ω is an open domain in $\mathbb{R}^n$). In particular, it follows that $\|t^{-k/n}(f^{**}(t)-f^{*}(t))\|_{L^p} \le \|D^k f\|_{L^p}$ for each $f \in W_0^{k,p}(\Omega)$ if $1 < p \le n/k$. The expression on the left is the "norm" in the Lorentz space $L(q,p)$ with $\frac{1}{q} = \frac{1}{p} - \frac{k}{n}$, surely for $q < \infty$ and as a matter of a convention for $q = \infty$. In fact, it is not a true norm; this explains the fact that in the limiting case $p = n/k$, the new inequality is strictly sharper than the embedding obtained by Brezis-Wainger and Hansson (which is known to be optimal in the r.i. class).

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