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Commutators for the maximal and sharp functions. (English)

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Let

$$M_p f(x) = \sup_{x\in Q}\Big(|Q|^{-1}\int_Q |f|^p\Big)^{1/p}$$

be the Hardy-Littlewood maximal function, where $1 \leq p < \infty$ and Q denotes a cube with sides parallel to the coordinate axes. The sharp function is defined by

$$f^{\#}(x) = M^{\#}f(x) = \sup_{x\in Q}|Q|^{-1}\int_Q |f - f_Q|,$$

where f_Q is the average of f over Q. If Q_0 is a fixed cube, the Hardy-Littlewood maximal function related to Q_0 is defined by

$$M_{p,Q_0}(f)(x) = \sup_{x\in Q\subseteq Q_0}\Big(|Q|^{-1}\int_Q |f|^p\Big)^{1/p}.$$

The authors proved the following two results.

Proposition 4. Let b be a real valued, locally integrable function on $\mathbb{R}^n$. The following assertions are equivalent:

(i) The commutator $[M_p, b]$ is bounded in L^q, for all $q \in (p,\infty)$.

(ii) The commutator $[M_p, b]$ is bounded in L^q for some $q \in (p,\infty)$.

(iii) B is in BMO and b^- is in L^∞.

(iv) There exists $q \in [1,\infty)$ such that $\sup_Q |Q|^{-1}\int_Q |b - M_{p,Q}(b)|^q < \infty$.

(v) $\sup |Q|^{-1}\int_Q |b - M_{p,Q}(b)|^q < \infty$ for all $q \in [1,\infty)$.

Proposition 6. Let b be a real valued, locally integrable function on $\mathbb{R}^n$. The following assertions are equivalent.

(1) $[M^{\#}, b]$ is bounded in L^q, for all $q \in (p,\infty)$.

(2) $[M^{\#}, b]$ is bounded in L^q for some $q \in (p,\infty)$.

(3) $b \in$ BMO and $b^- \in L^\infty$.

(4) There exists $q \in [1,\infty)$ such that $\sup_Q |Q|^{-1}\int_Q |b(x) - 2(b\chi_Q)^{\#}(x)|^q dx < \infty$.

(5) The inequality in (4) is true for all $q \in [1,\infty)$.

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46E30 Spaces of measurable functions

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