FLORIDA ATLANTIC UNIVERSITY

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PhD Dissertation Defense

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Formation, Evolution, and Breakdown of Invariant Tori in Dissipative Systems: from Visualization to Computer Assisted Proofs

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Advisors: Dr. Jason Mireles-James and Dr. Vincent Naudot

The goal of this work is to study smooth invariant sets using high order approximation schemes. Whenever possible, existence of invariant sets are established using computer-assisted proofs. This provides a new set of tools for mathematically rigorous analysis of the invariant objects. The dissertation focuses on application of these tools to a family of three-dimensional dissipative vector fields, derived from the normal form of a cusp-Hopf bifurcation. The vector field displays a Neimark-Sacker bifurcation giving rise to an attracting invariant torus. We examine the torus via parameter continuation from its appearance to its breakdown, scrutinizing its dynamics between these events. We also study the embeddings of the stable/unstable manifolds of the hyperbolic equilibrium solutions over this parameter range. We focus on the role of the invariant manifolds as transport barriers and their participation in global bifurcations. We then study the existence and regularity properties for attracting invariant tori in three-dimensional dissipative systems of ordinary differential equations and lay out a constructive method of computer assisted proof which pertains to explicit problems in non-perturbative regimes. We get verifiable lower bounds on the regularity of the attractor in terms of the ratio of the expansion rate on the torus with the contraction rate near the torus. We look at two important cases of rotational and resonant tori. Finally, we study the related problem of approximating two-dimensional subcenter manifolds of conservative systems. As an application, we compare two methods for computing the Taylor series expansion of the graph of the subcenter manifold near a saddle-center equilibrium solution of a Hamiltonian system.

Please contact Dr. Hongwei Long (hlong@fau.edu) for an electronic copy of the dissertation.
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