An excursion through mathematics and its history (and some trivia)

MATH DAY 2015—TEAM COMPETITION

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A quick review of the rules

- History (or trivia) questions alternate with math questions
- Math questions are numbered by MQ1, MQ1, etc. History questions by HQ1, HQ2, etc.
- Math answers should be written on the appropriate sheet of the math answers booklet.
- The answer to every math question will be either an integer (mostly positive) or the square root of a positive integer.
- Square roots of positive integers MUST be entered in the form $m\sqrt{n}$ where m, n are positive integers and n is square free. Examples: $\sqrt{12}$ should be entered as $2\sqrt{3}$, $\sqrt{50}$ should be entered as $5\sqrt{2}$, $3\sqrt{98}$ as $21\sqrt{7}$.
- History questions are multiple choice, answered using the clicker.
- Math questions are worth the number of points shown on the screen when the runner gets your answer sheet. That equals the number of minutes left to answer the question.
- Have one team member control the clicker, another one the math answers booklet

Rules--Continued

- All history/trivia questions are worth 1 point.
- The team with the highest math score is considered first. Next comes the team with the highest overall score, from a school different from the school of the winning math team. Finally, the team with the highest history score from the remaining schools.

HQ0-Warm Up, no points

- Cryptography is a hot research area at FAU, involving several departments. Cryptography is so called because

A. It is developed in crypts.

B. It was invented by a grumpy old man called Nick Cryptographiakis.

C. It is Greek for secret writing.

D. It was invented by aliens from the planet Krypton.

E. Nobody knows why it is so called.

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20 seconds

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Time’s Up!

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Demonstrating the points system

- For math questions there will be a number in the lower right corner. It will change every minute. Here I am illustrating with numbers changing every 10 seconds. Try to imagine 10 seconds is a minute. The first number tells you the maximum number of points you can get for the question. Assume a question is on the screen.

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1

I may/will give less than 4 minutes if I see everybody is in, or I think the question is too easy.

TIME's UP!

THE CHALLENGE BEGINS

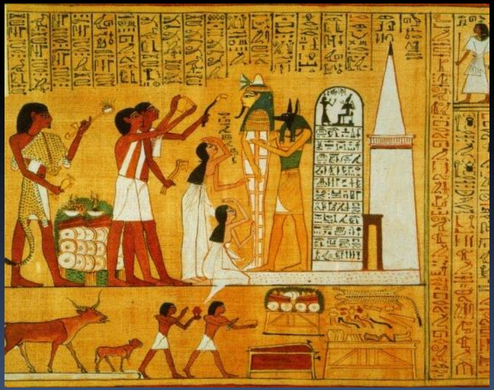
VERY IMPORTANT!

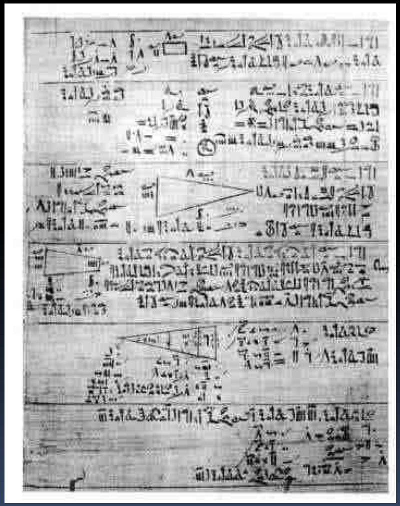
Put away all electronic devices; including calculators.
Mechanical devices invented more than a hundred years ago, are OK.

HQ1. In Old Egypt

In the Rhind papyrus (c. 1650 BCE) Ahmes the scribe uses a method known as “false position” to solve some problems. The method of false position solves a problem by

A. Working backwards from the solution.
B. Making a false assumption and reaching a contradiction.
C. Guessing the solution, and then adjusting.
D. Obtaining only an approximate solution.
E. Solving a completely different problem.





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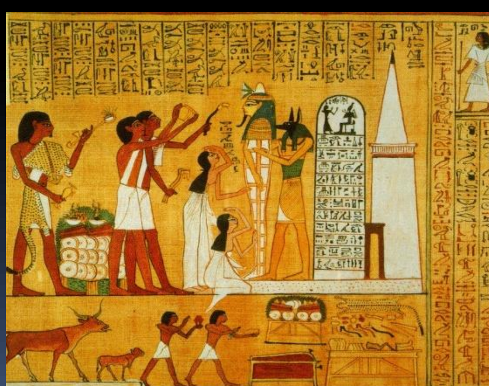

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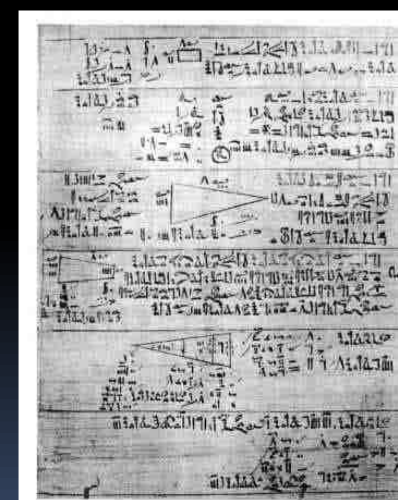

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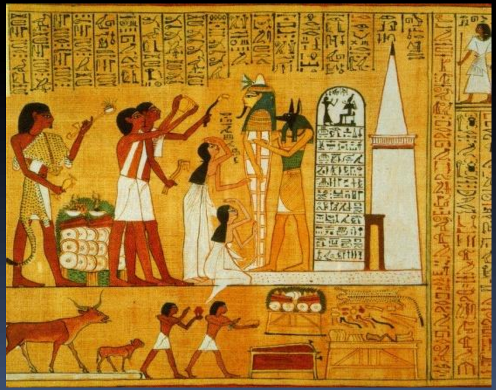

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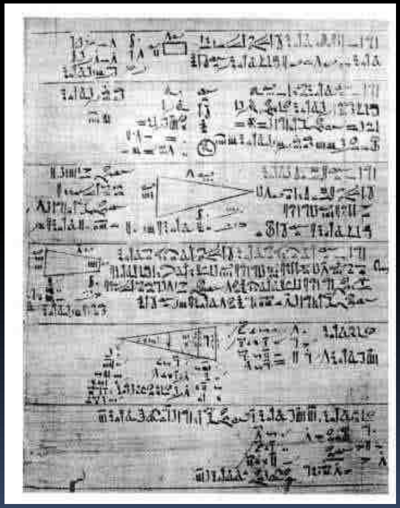

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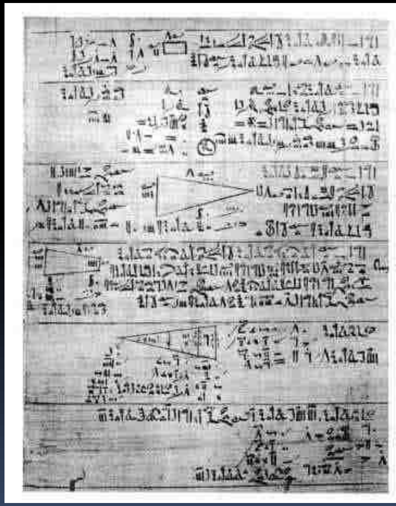
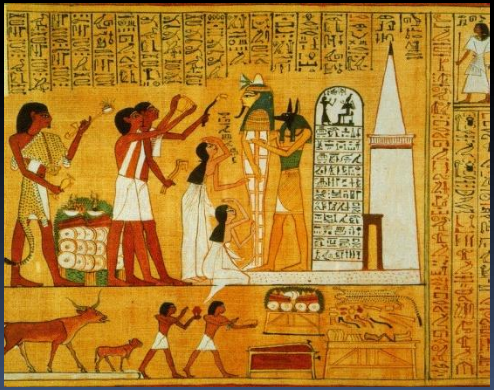
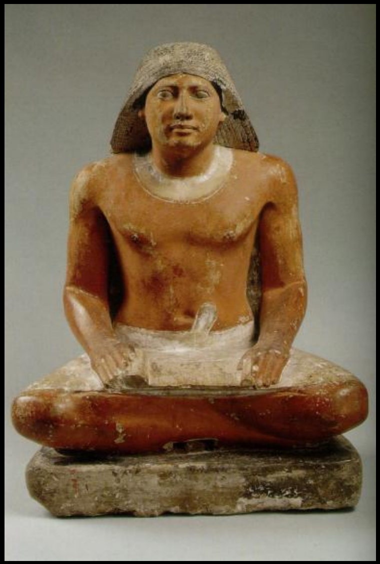




Time's Up!

MQ1. Egyptian Pi

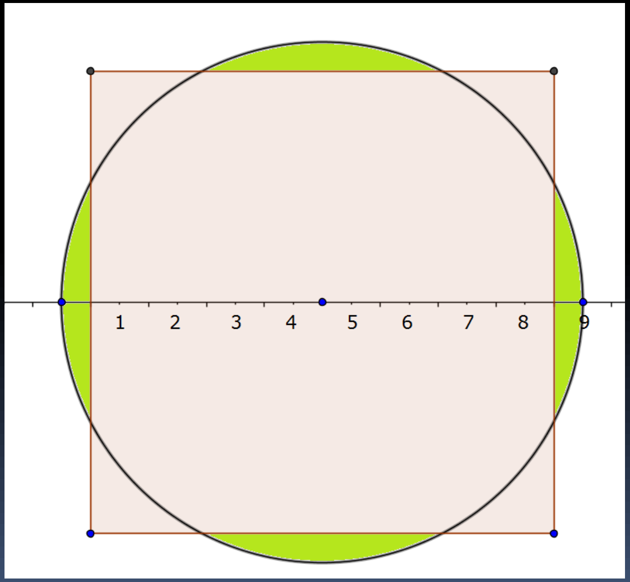


In the Rhind papyrus Ahmes, the scribe, writes:

Cut off 1/9 of a diameter and construct a square upon the remainder; this square has the same area as the circle.

This would be correct if $\pi = m/n$ where m, n are some specific positive integers with no common factors.

What is $m + n$?



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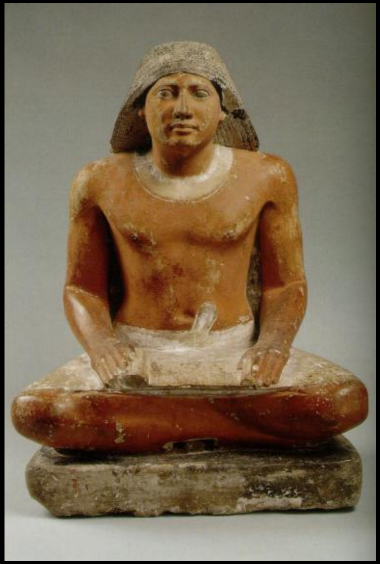

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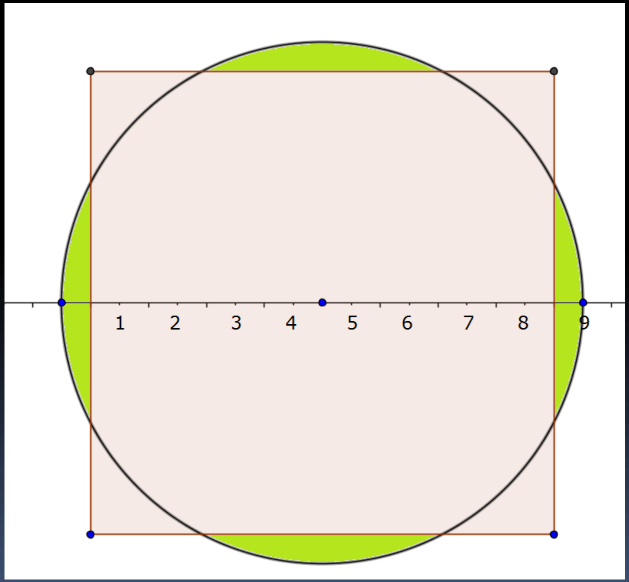

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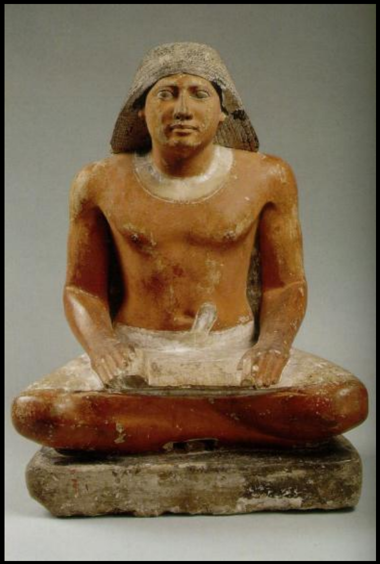

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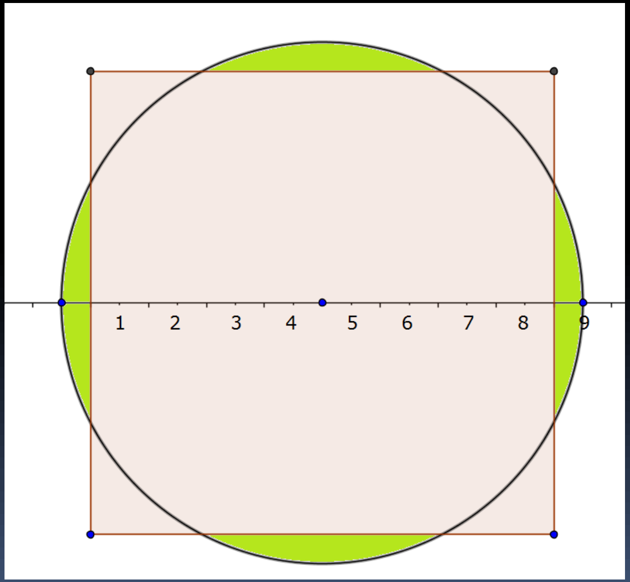

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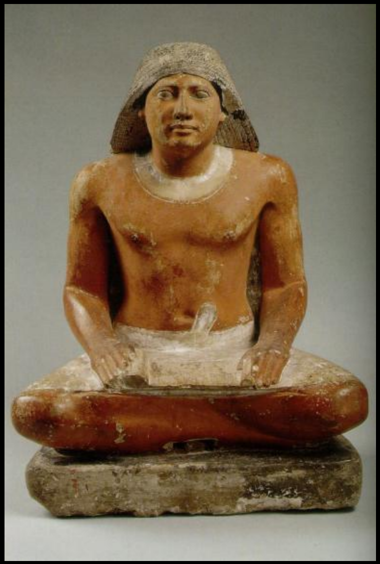

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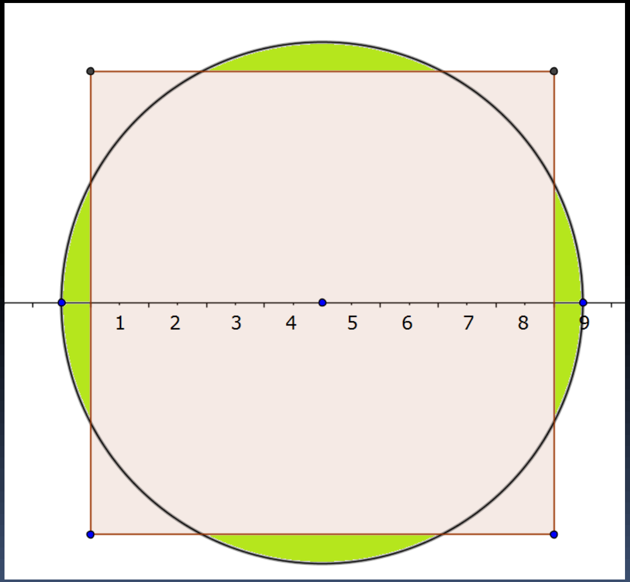

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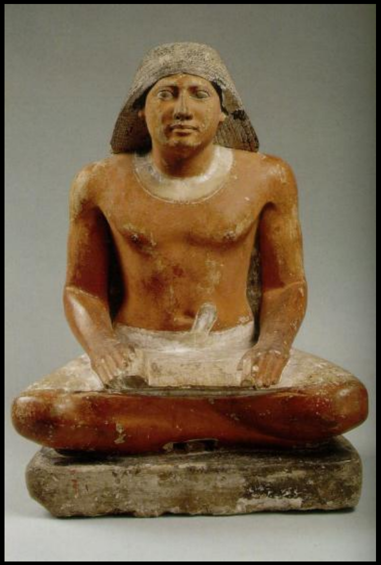

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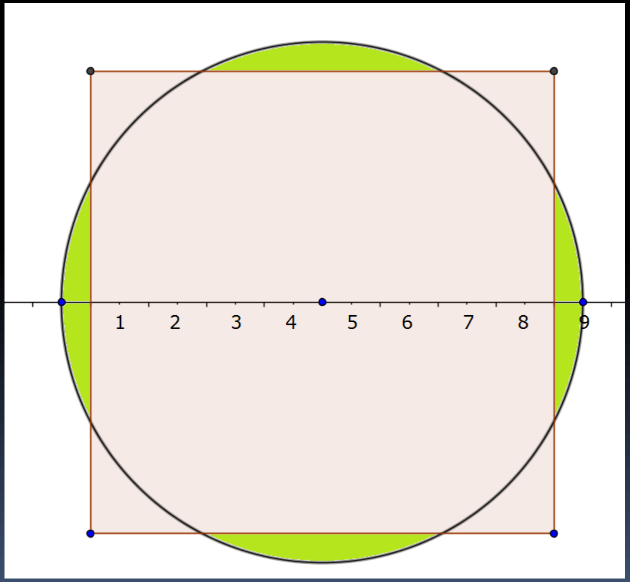

TIME's UP!

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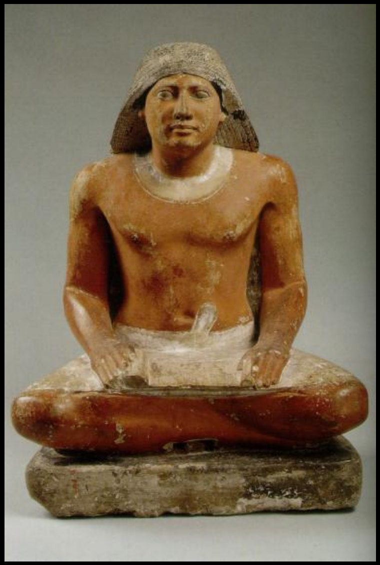

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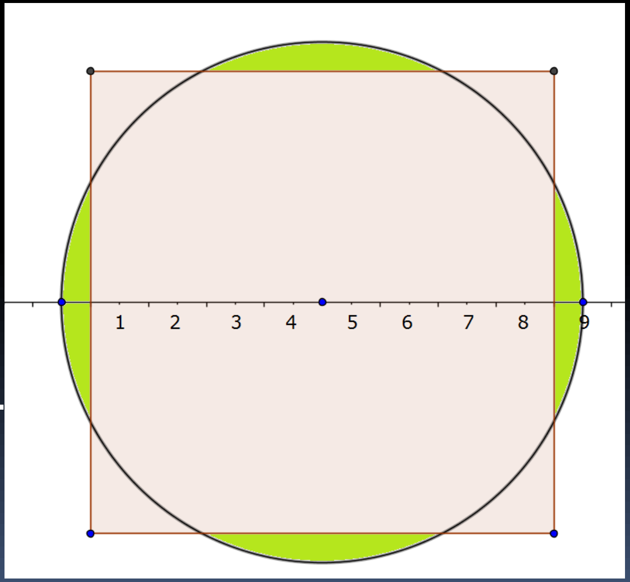
The Egyptian π works out to $\pi = 256/81 = 3.16049\ldots$

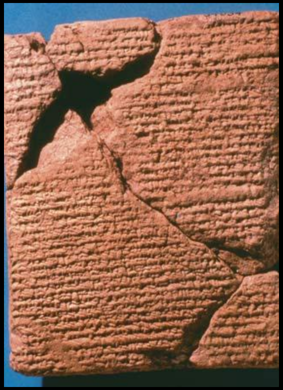


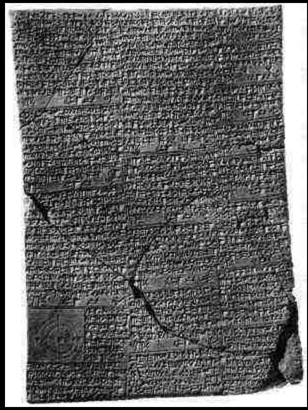
m + n = 337

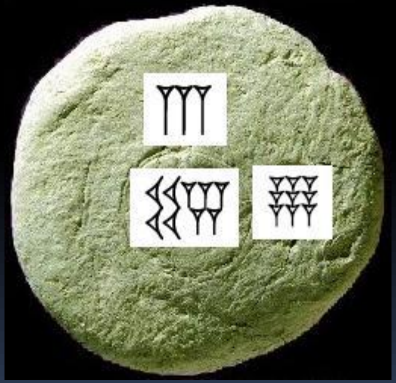
HQ2. Babylon (1800BCE-500BCE)

The Babylonians were perhaps the first to introduce a positional number system. What number did they use as a base?

A. 5
B. 10
C. 20
D. 60
E. 100







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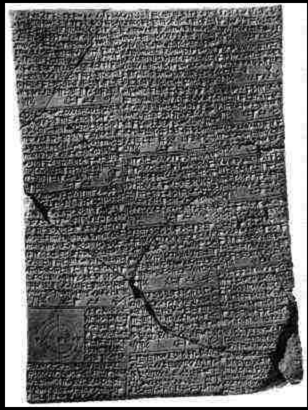

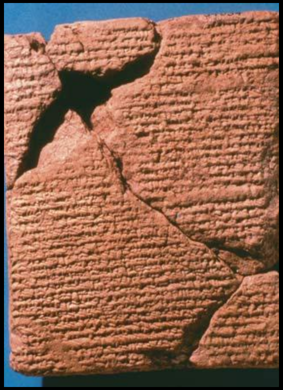

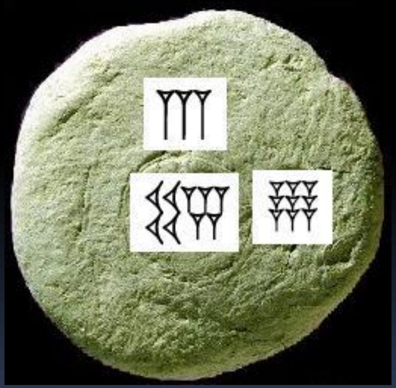

20 seconds

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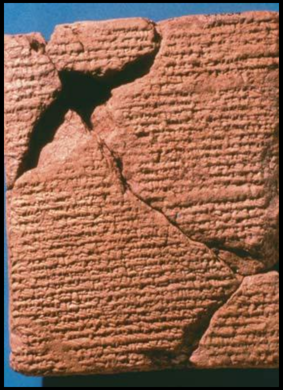

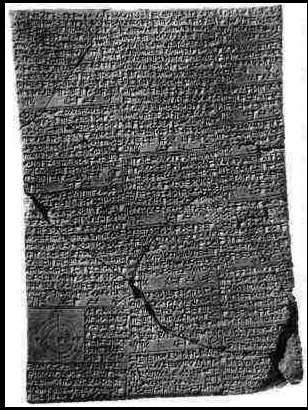

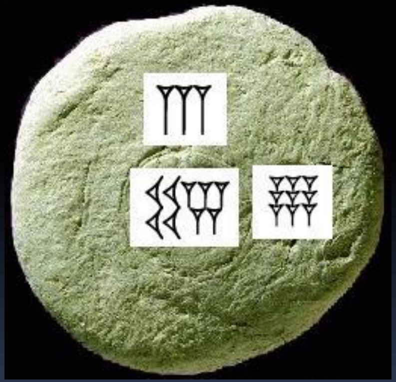

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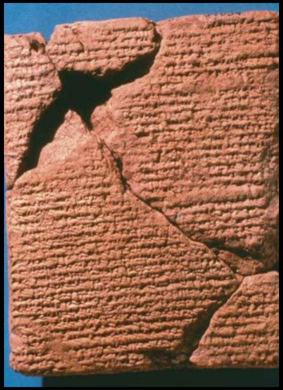
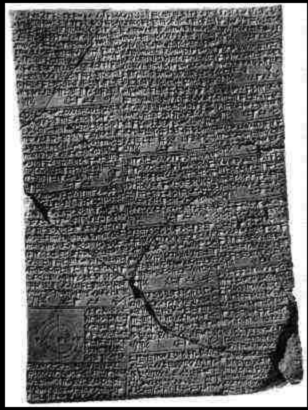
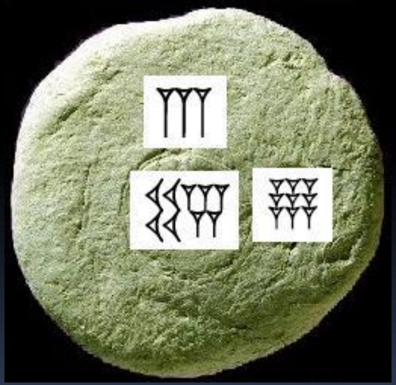


Time's Up!

MQ2. Babylonian Equations

A tablet found in Susa in 1936 asks for the sides x, y of a rectangle given

$$xy = 24, \quad x^3 d = 540$$

where d is a diagonal of the rectangle. (I modified the numbers a bit to make it more exciting)

$$\boxed{\text{FIND } x.}$$

Numbers get large before getting small.
Here are some computations you might find helpful:

$$\begin{aligned} 54^2 &= 2916 \\ 576^2 &= 331776 \\ 1224^2 &= 1498176. \end{aligned}$$

How many miles to Babylon?
Three score and ten.
Can I get there by candle-light?
Yes, and back again.
If your heels are nimble and light,
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SOLUTION. $d = \sqrt{x^2 + y^2} = \sqrt{x^2 + (24/x)^2} = (\frac{1}{7}x)\sqrt{x^4 + 576}$;
$540 = x^3 d = x^2\sqrt{x^4 + 576}$, squaring, etc., $x^8 + 576x^4 - 291600 = 0$. Solve for x^4.
$x^4 = (-576 \pm \sqrt{1498176})/2 = (-576 \pm 1224)/2$; discard the negative root so $x^4 = 324$. $x^2 = 18$, $x = \sqrt{18}$.

$$\boxed{x = 3\sqrt{2}.}$$

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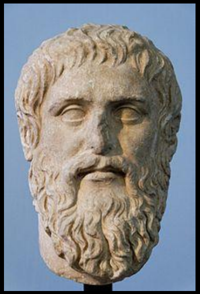
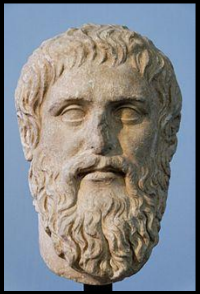
HQ3. Plato's Principles



About 387 BCE, Plato, one of the greatest thinkers of all times, founded a school. The story goes that on its entrance was written "Let none but geometers enter." The school became known as

A. The Academy
B. The Athenaeum
C. The Collegium
D. The Alexandrium
E. The University

HQ3. Plato’s Principles



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20 seconds

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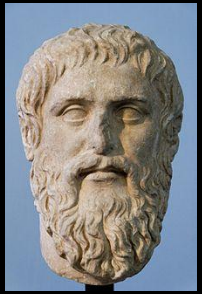

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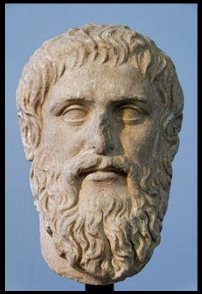

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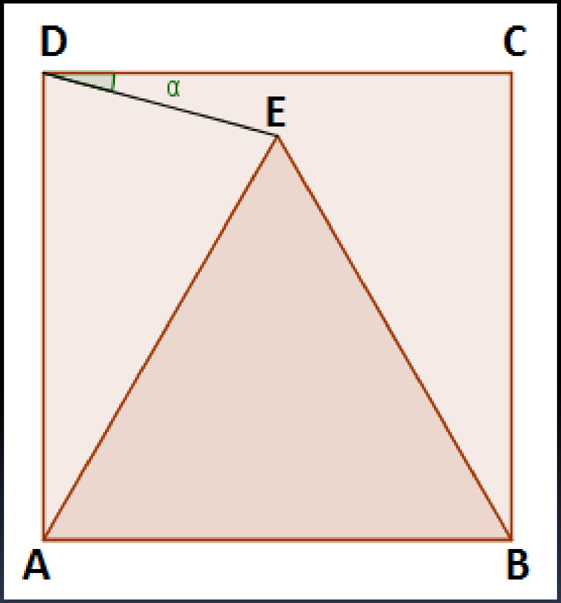
Ancient Road to the Academy

MQ3-What's the Angle?

- The triangle AEB inside the square ABCD is equilateral. Find the measure **in degrees** of the angle α = EDC.



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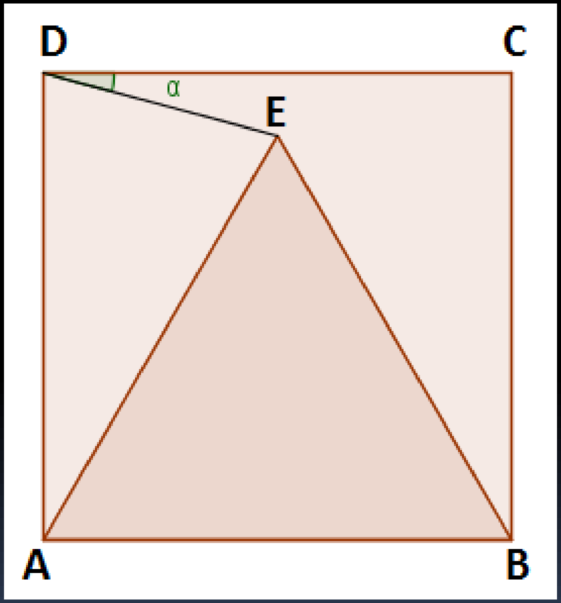

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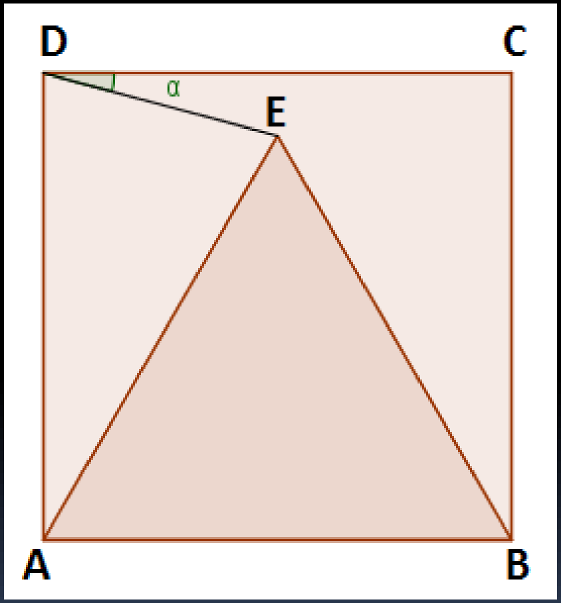

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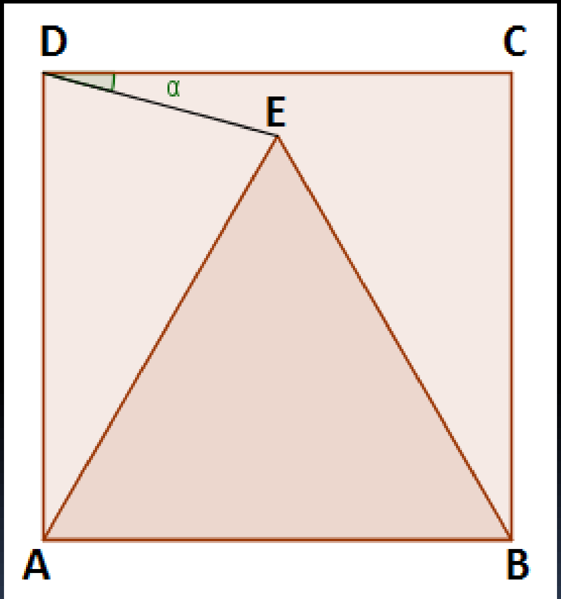

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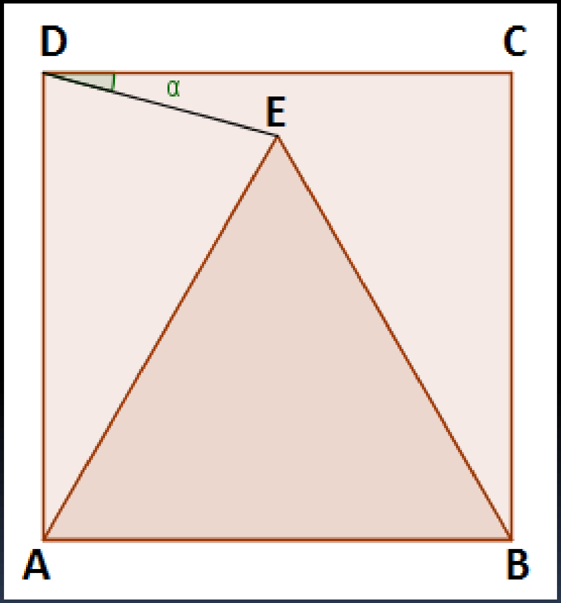

1

TIME’s UP!

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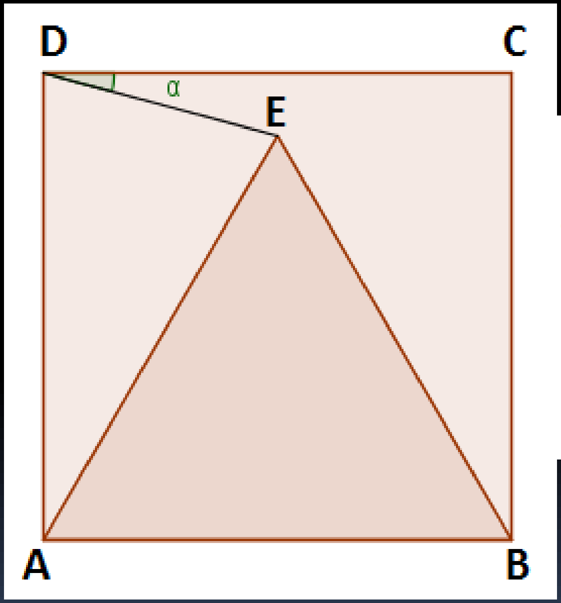

Because $\triangle AEB$ is equilateral, $\angle BAE = 60^o$, thus $\angle EAD = 30^o$. Because $\triangle AEB$ is equilateral, $AE = AD$ and $\triangle AED$ is isosceles, $\angle AED = \angle ADE$. Since $\angle EAD + AED + ADE = 180^o$, we see that $\angle ADE = 75^o$. Thus $\alpha = 90^o - 75^o = 15^o$

$\boxed{15}$

HQ4. Cubic Man

He may have been the first person to systematically try to solve cubic equations, which he did by geometric means. Mathematician, astronomer and poet, he was

A. Al Khwarizmi
B. Omar Khayyam
C. Harun al Rashid
D. Averroes
E. Ali Baba

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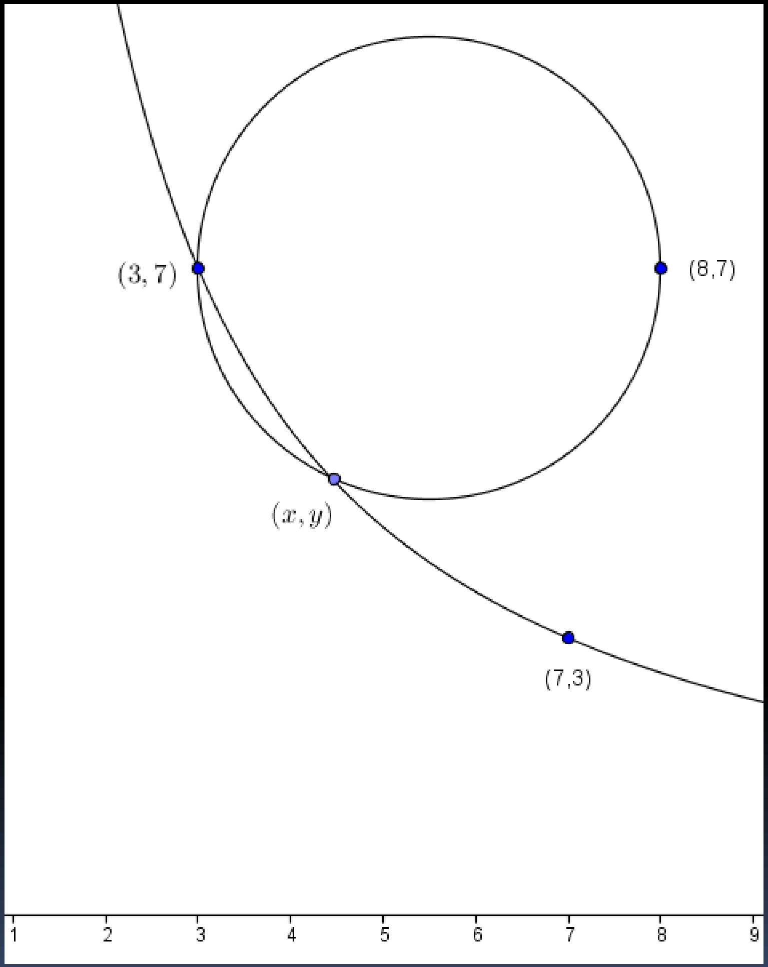
Ghiyath al-Din Abu'l-Fath Umar ibn Ibrahim Al-Nisaburi **al-Khayyami**

MQ4. Cubic Questions



The picture shows a hyperbola of equation $xy = c$ intersecting a circle of diameter 5. Four points are marked. Of three you are given the coordinates. The abscissa x of the fourth satisfies a cubic equation of the form $x^3 + mx = px^2 + q$.

Find q.

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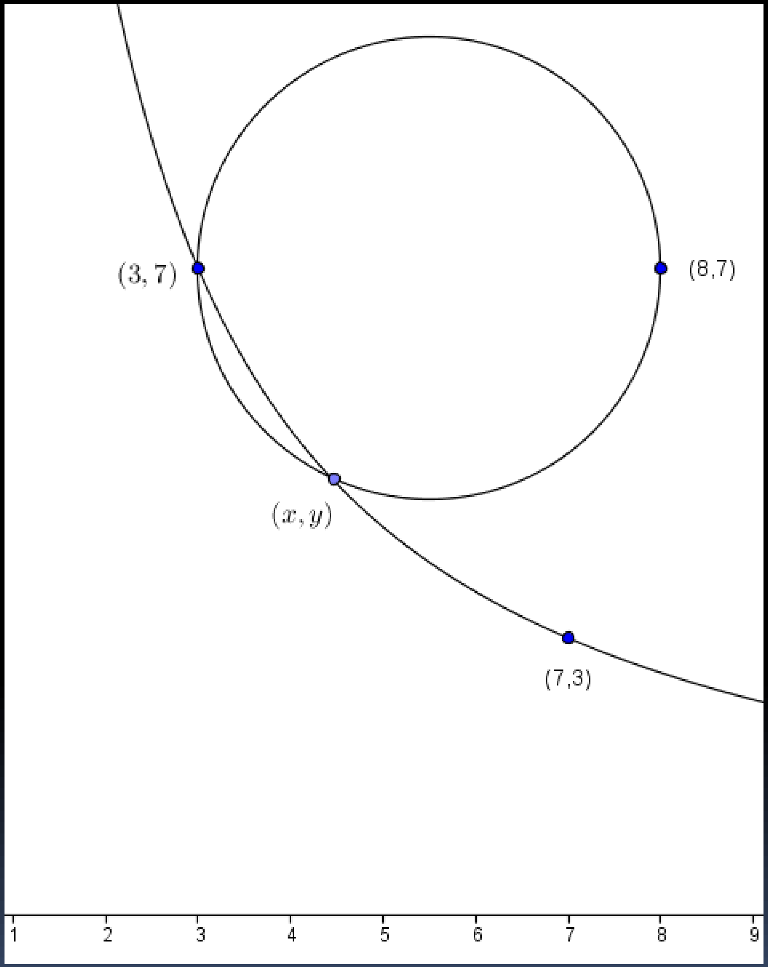

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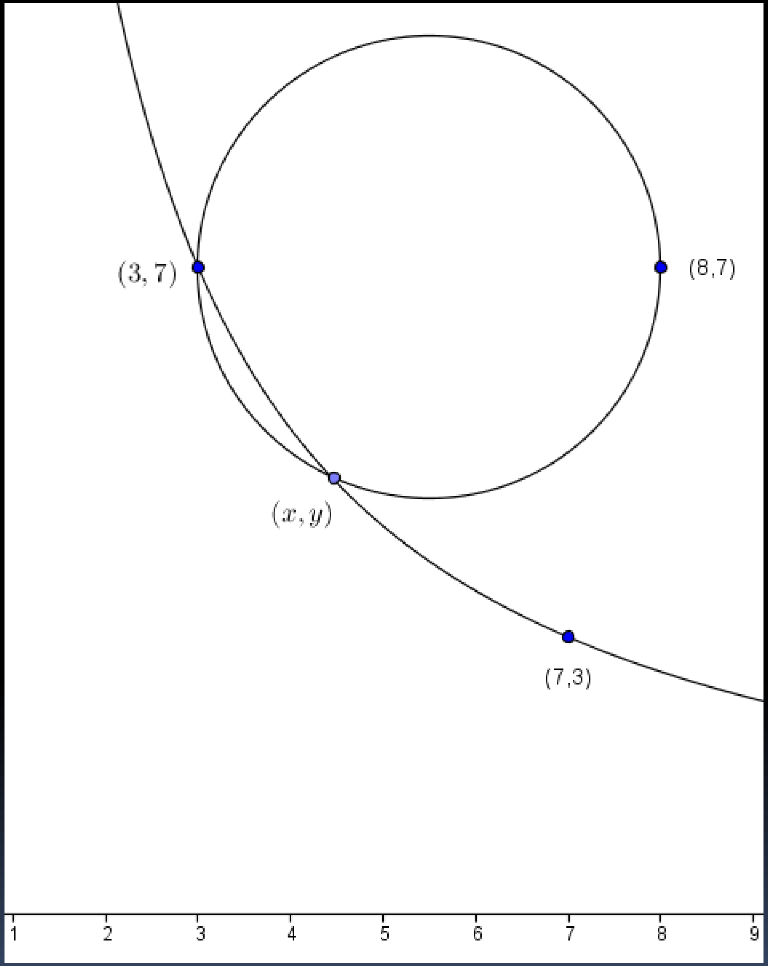

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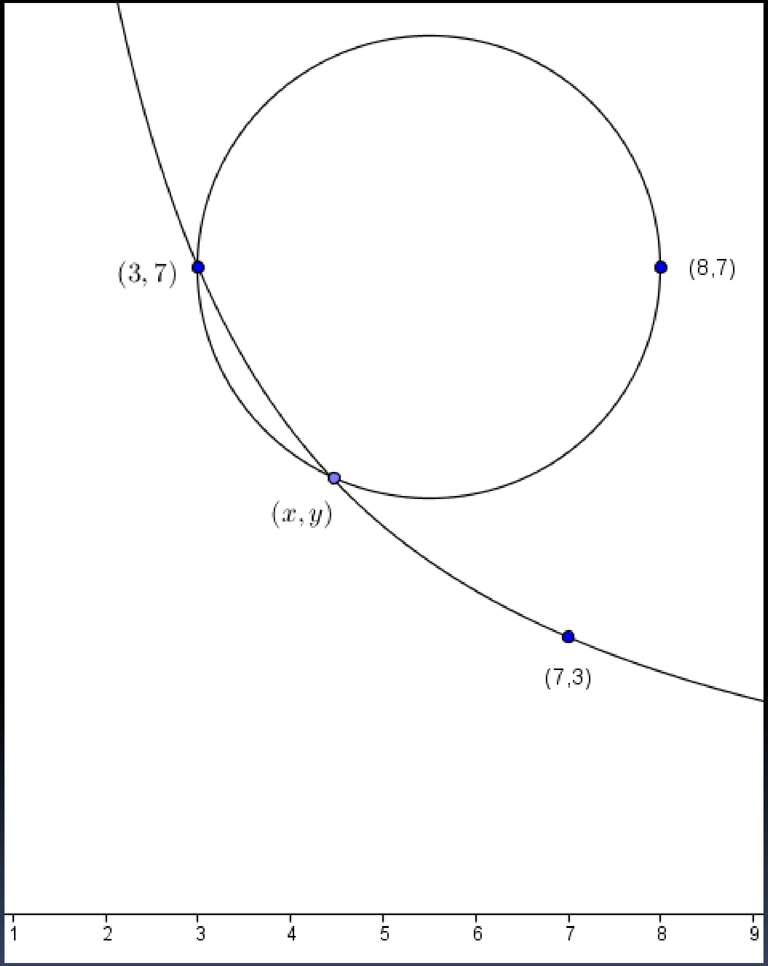

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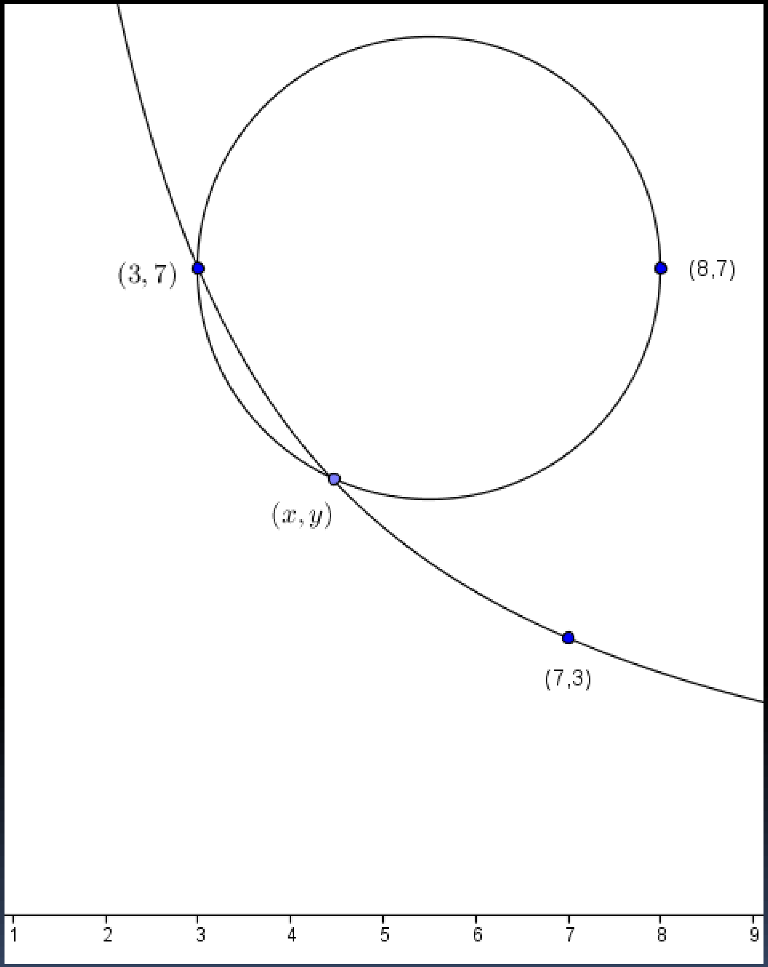

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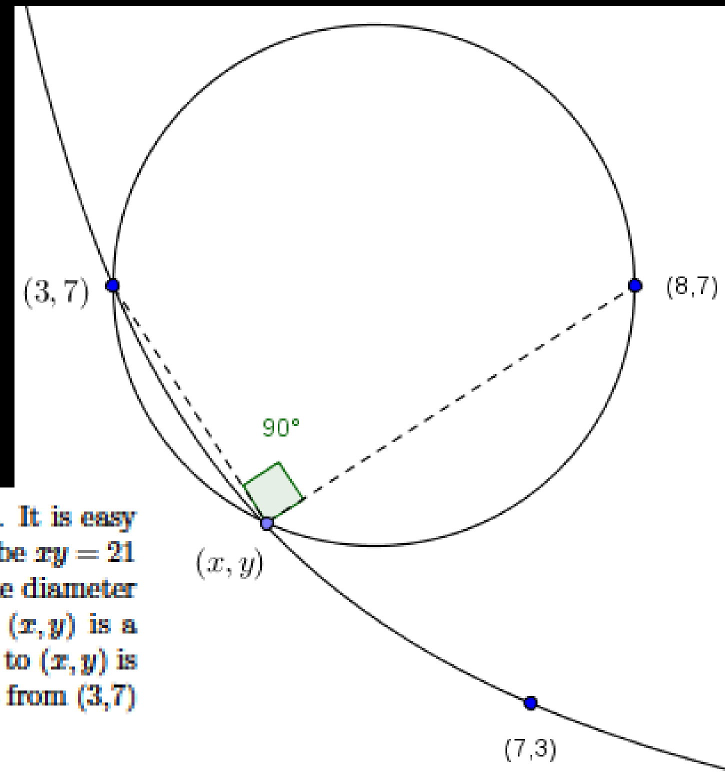

This is probably the easiest way of solving this. It is easy to see that the equation of the hyperbola must be $xy = 21$ or $y = 21/x$. Since points $(3,7), (8,7)$ are on the diameter of the circle, the angle formed with the point (x, y) is a right angle; the slope of the segment from $(8,7)$ to (x, y) is minus the reciprocal of the slope of the segment from (3,7) to (x, y). Thus

$$\frac{y-7}{x-3} = -\frac{x-8}{y-7}, \quad \text{so } (y-7)^2 = -(x-8)(x-3).$$

Replacing y by $21/x$ and using that $21/x - 7 = 7(3-x)/x =$ we get (after canceling one $(x-3)$ and multiplying by x^2) the equation $49(x-3) = -x^2(x-8)$, which rearranges to $x^3 + 49x = 8x^2 + \boxed{147}$.

HQ5. The Sky's the Limit

Claudius Ptolemy, born in the year 85 (approx.) wrote one of the most influential mathematics/astronomy books ever. For over 1500 years it was the astronomical standard. The book was called *The Mathematical Compilation* but it became better known by its Arab name, which was

A. The Algebricon.
B. The Almagest.
C. The Alembic.
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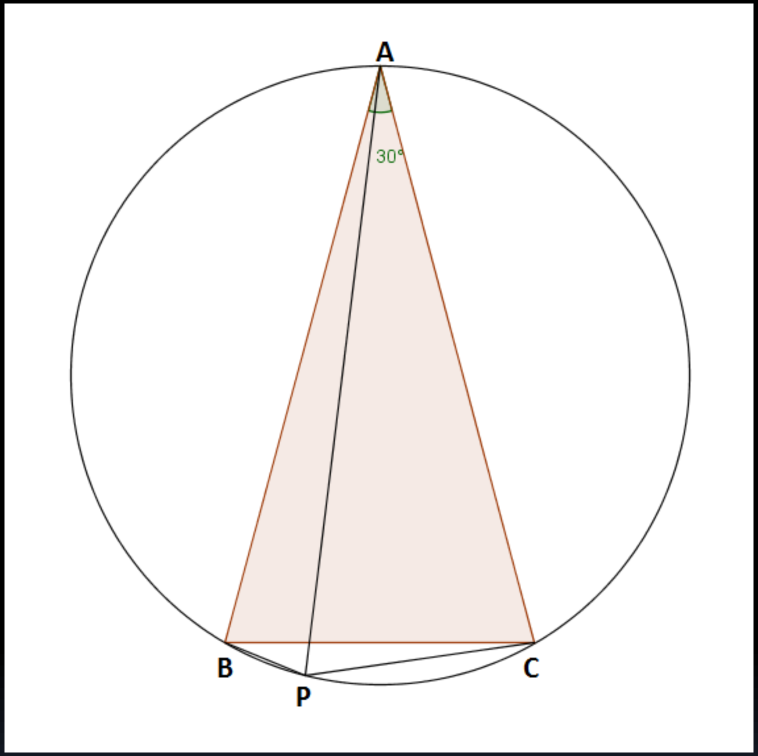
Time's Up!

MQ5. Encircled Isosceles



The isosceles triangle ABC, of equal sides AB, AC, has an angle of 30 degrees at A. A line from A intersects the circumcircle at P.

Let $s = \frac{|AP|}{|BP|+|PC|}$. Then $s^2 = m + \sqrt{n}$, m, n are positive integers. Find

$m + n$

Some helpful formulas

$$\sin 30^\circ = \frac{1}{2}$$
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$
$$\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$$
$$\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Not all of these are needed.

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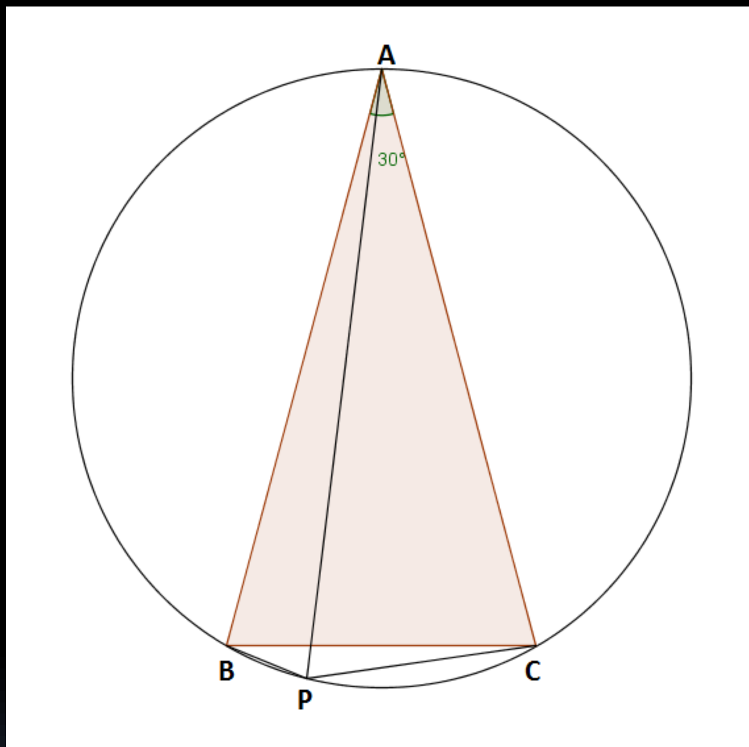

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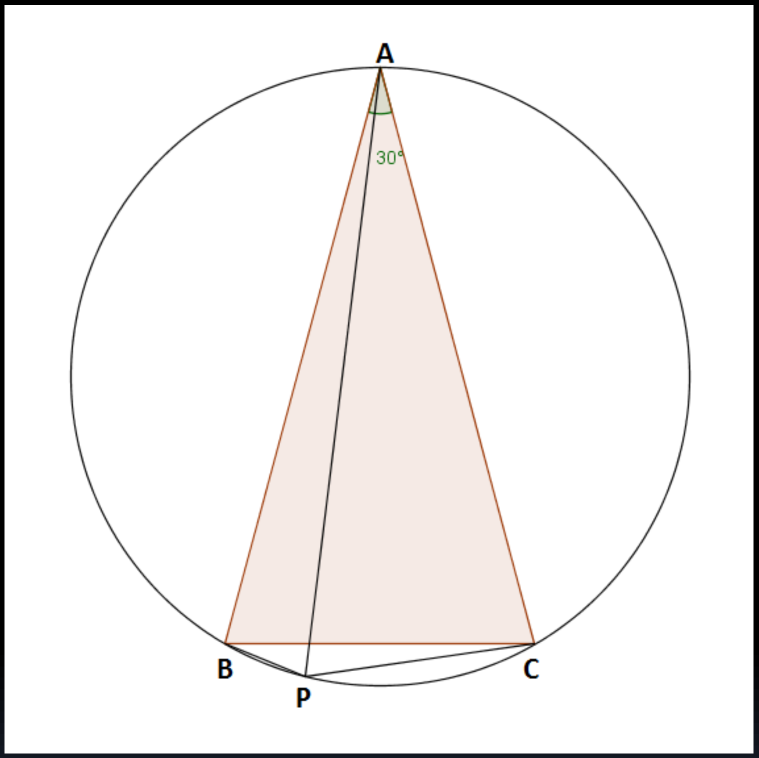

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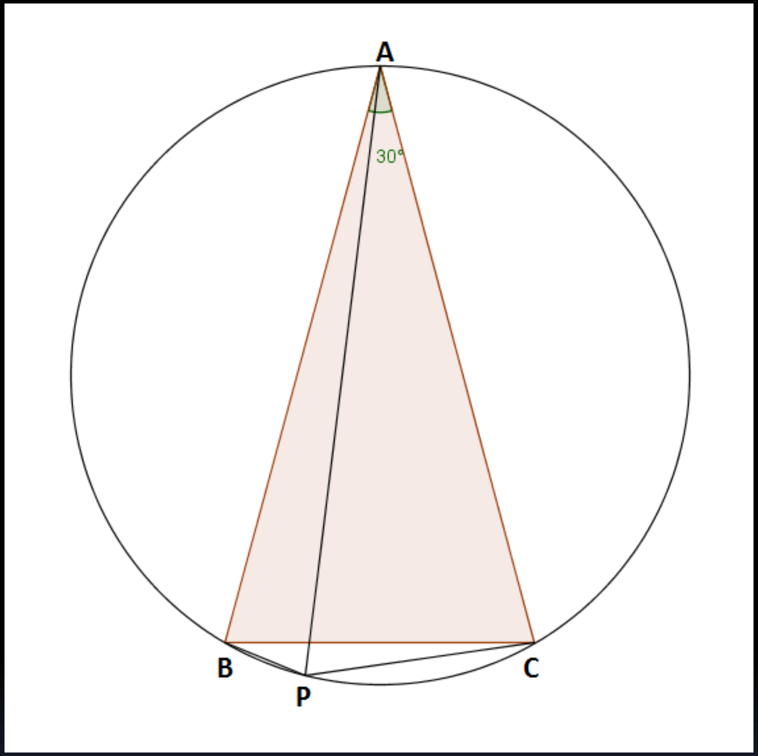

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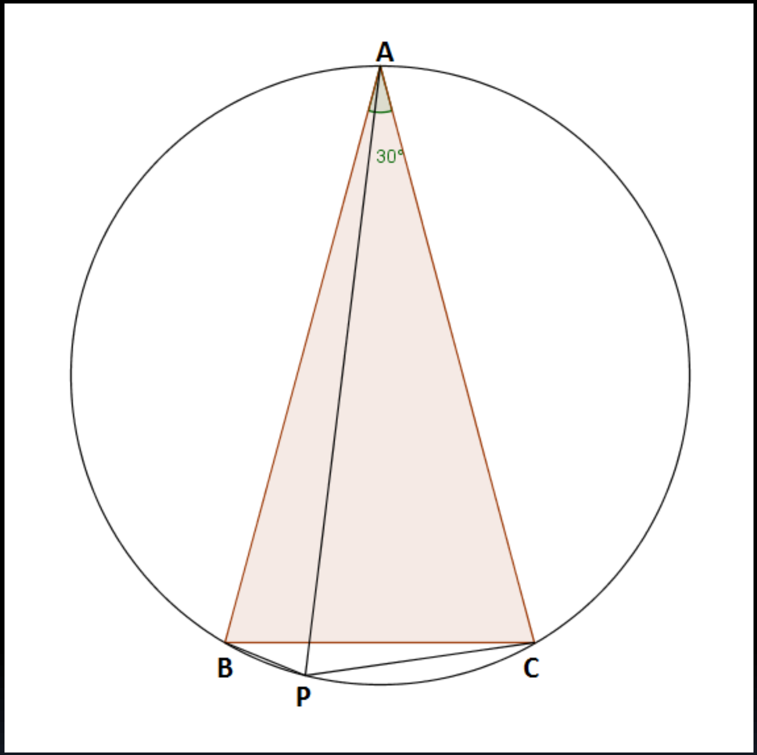

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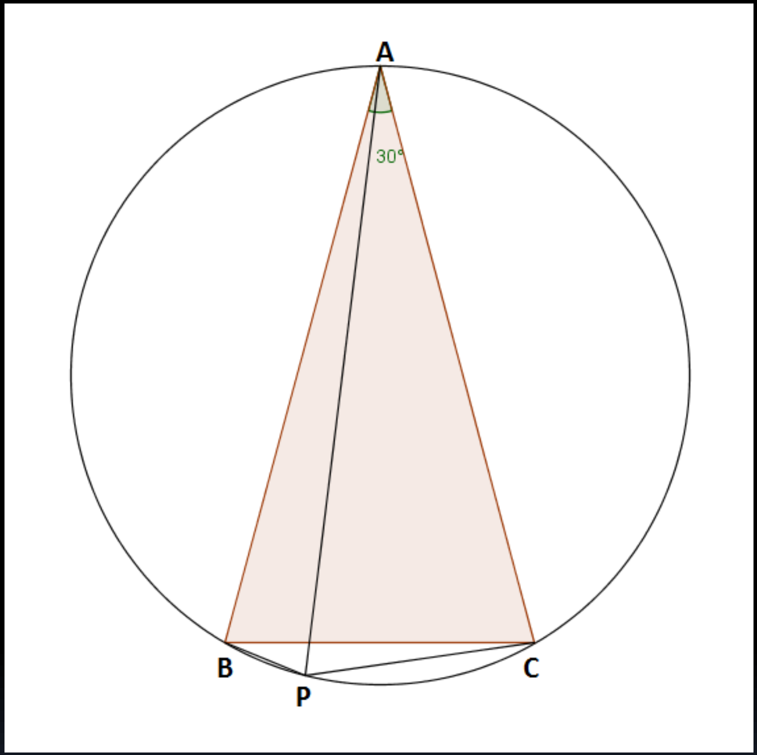

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By Ptolemy's Theorem applied to the quadrilateral $ABPC$, $|AP| \cdot |BC| = |AC| \cdot |BP| + |AB| \cdot |PC|$. Since $|AB| = |AC|$, this gives $\frac{|AP|}{|BP| + |PB|} = \frac{|AB|}{|BC|}$. By the law of sines, $|BC| = 2R \sin 30° = R$, $|AB| = 2R \sin 75° = 2R \cos 15° = (\sqrt{6} + \sqrt{2})R/2$. Thus $s = (\sqrt{6} + \sqrt{2})/2$, $s^2 = 2 + \sqrt{3}$.

$$m = 2, n = 3, m + n = 5$$

HQ6. Marginal Notes

Pierre de Fermat wrote the famous lines "*To…divide any power whatever into two powers of the same denomination above the second is impossible, and I have assuredly found a marvelous proof of this, but the margin is too narrow to contain it*" on the margin of what book?

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20 seconds

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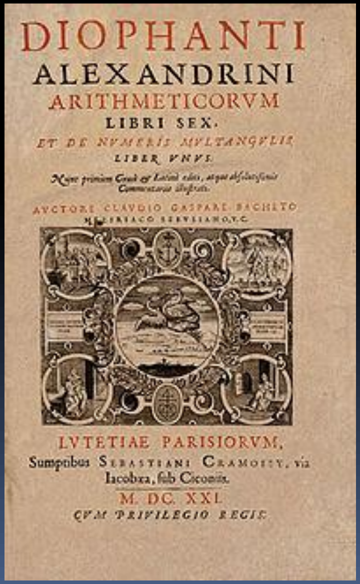
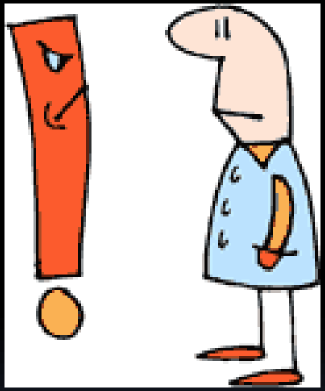


MQ6. Numerous Numbers

How many positive integers $n \leq 50$ have the property that n does **NOT** divide $(n-1)!$?



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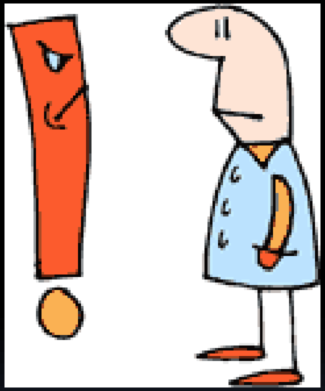

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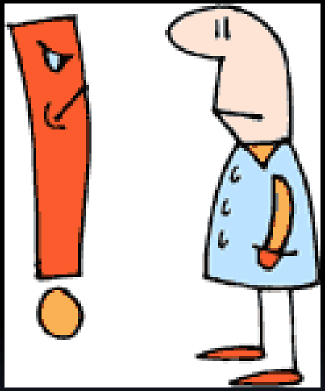

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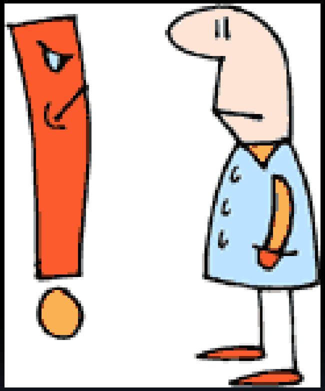

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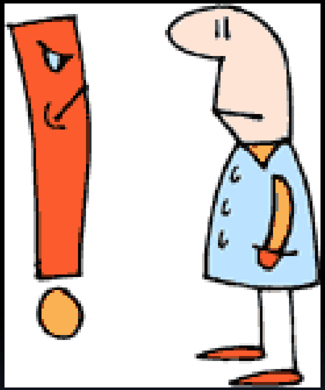
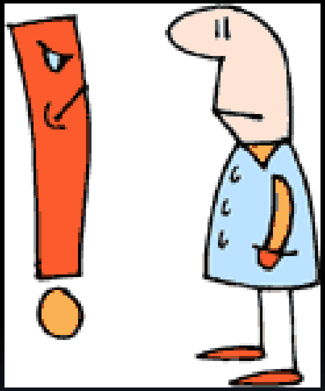


1

TIME’s UP!

MQ6. Numerous Numbers

How many positive integers $n \leq 50$ have the property that n does **NOT** divide $(n-1)!$?
SOLUTION: 4 and all the prime numbers ≤ 50: 2, 3, 4, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.



16 numbers in all.

HQ7. The Dawn of Newton

Which of the following is NOT the title of a book or article written by Sir Isaac Newton:

A. The Chronology of Ancient Kingdoms
B. The Wave Theory of Light.
C. Philosophiae Naturalis Principia Mathematica
D. Opticks
E. Hypothesis explaining the properties of light

Nature and Nature's laws lay hid in night,
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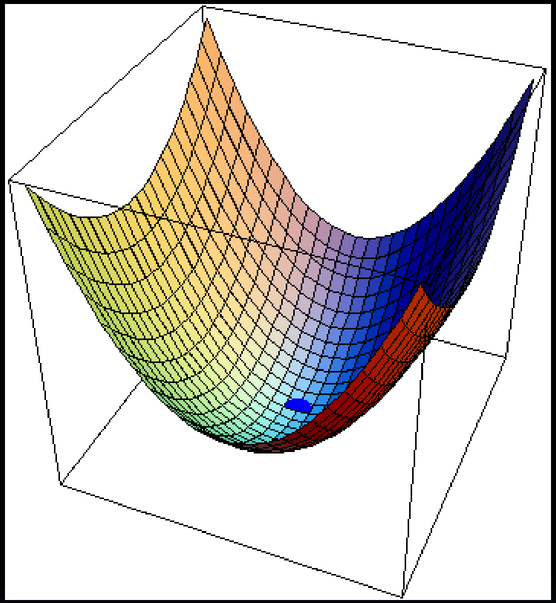
MQ7. Minimalist Dreams

What is the minimum value of

$$(a-b)^2 + \left(\sqrt{18-a^2} - \frac{49}{b}\right)^2$$

for real numbers a, b , $0 \le a \le \sqrt{18}$, $b > 0$.



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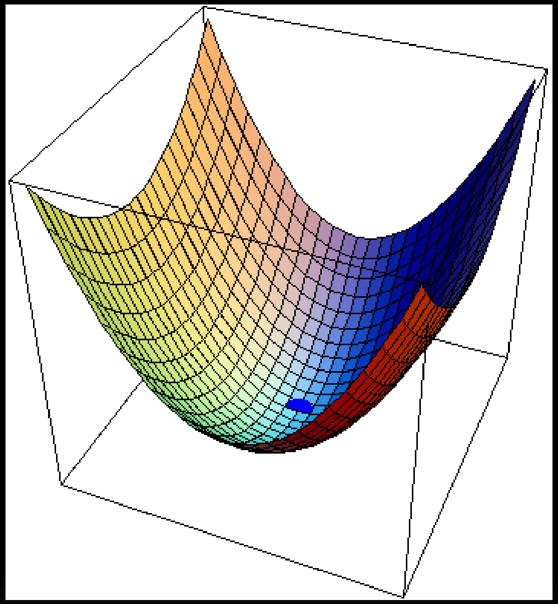

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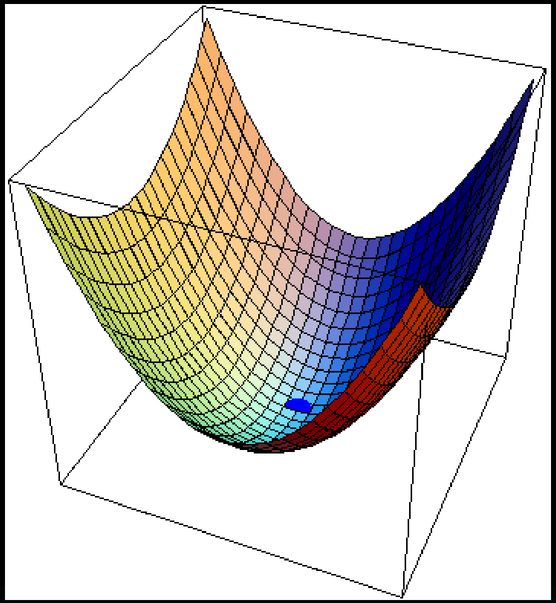

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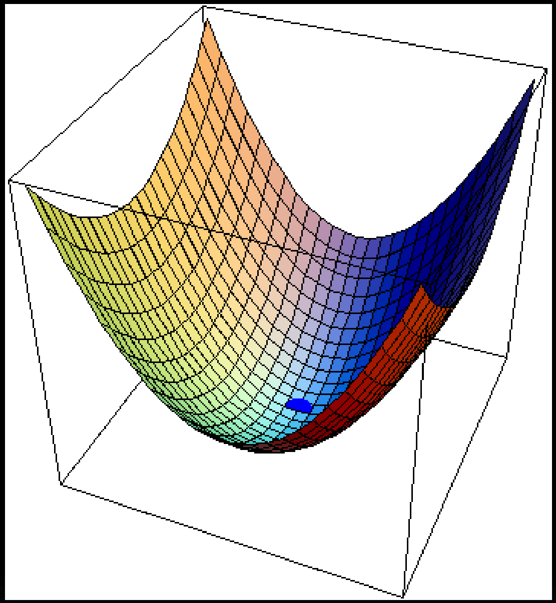

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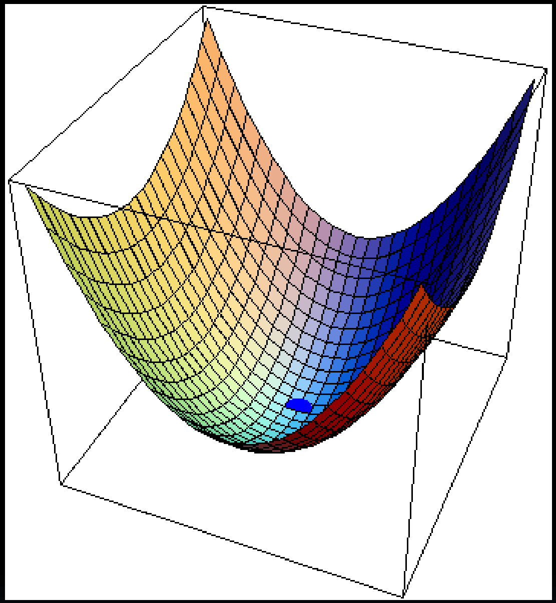

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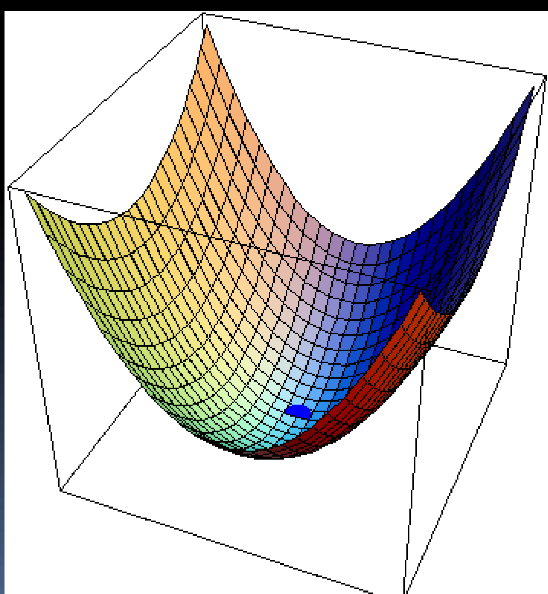
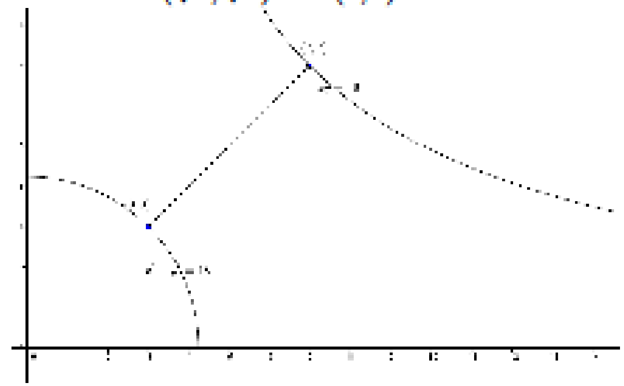


The expression to be minimized can be interpreted as the square of the distance between a point of coordinates $(a, \sqrt{18-a^2})$, so a point on the circle of radius $\sqrt{18}$, centered at $(0,0)$; and a point of coordinates $(b, 49/b)$, which is a point on the hyperbola of equation $xy = 49$. The picture below shows that the minimum is achieved when both points are on the line $x = y$, so the minimum is the square of the distance between the points of coordinates $(\sqrt{3}, \sqrt{3})$ and $(7, 7)$.



The answer is

$$\boxed{32}$$

HQ8. Euler the Great

Leonhard Euler (1707-1783) was without doubt the greatest mathematician and physicist of the 18th Century, and one of the greatest of all times. Which of he following is NOT true of Euler

A. He was a medical officer in the Russian Navy.
B. He laid the foundations for fluid dynamics.
C. He showed that exponential and trigonometric functions were closely related.
D. He laid the foundation for analytic number theory.
E. He proved Fermat's conjecture, that $2^{2^n}+1$ was prime for $n = 1,2,3,4,5$.

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20 seconds

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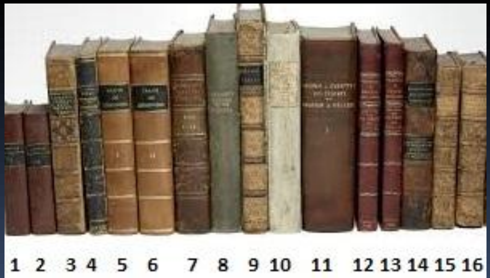
MQ8. Rare Books

Bibliophilus has 132 rare books in his library, each one in a slot numbered from 1 to 132. One morning he decides to reorder his books. He places every book that was in slots n for $1 \leq n \leq 66$ in slot $2n$; so the book in 1 goes to 2, the one in 2 goes to 4; the one in 66 to 132. This displaces all the books in slots 67 to 132, but opens all odd slots. He places the displaced books, in order, into the odd slots, so that 67 goes to 1, 68 to 3, 69 to 5, etc., 132 to 131. Next day he repeats the procedure. He repeats it the day after that. How many times can Bibliophilus rearrange books this way before having them again in their original order?



Partial View of the Library

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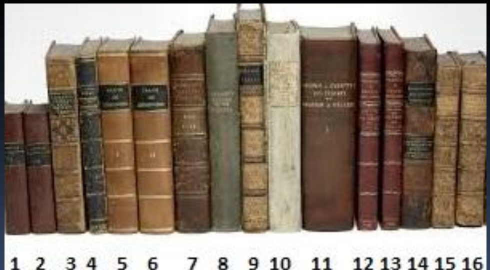

Partial View of the Library

4

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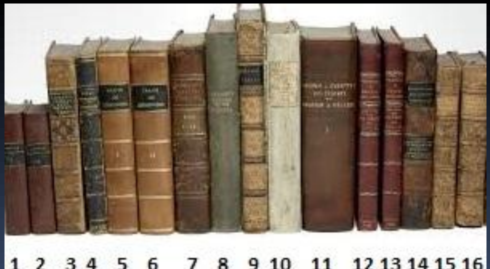

Partial View of the Library

3

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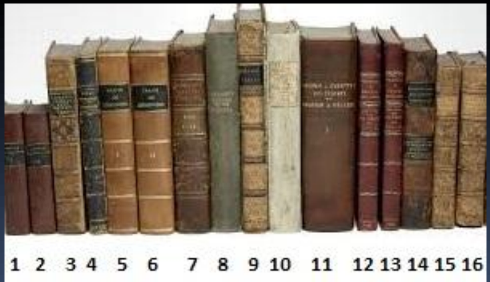

Partial View of the Library

2

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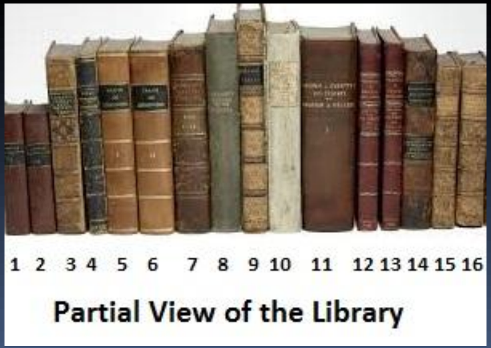

TIME’s UP!

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One sees that the rearrangement sends the book in position n to position m where $m \equiv 2n \pmod{133}$. So the question is equivalent to: What is the smallest positive integer k such that $2^k \equiv 1 \pmod{133}$? Computations can be shortened by knowing that $133 = 7 \times 19$, that thus $\phi(133) = \phi(7)\phi(19) = 6 \times 18 = 108$, where ϕ is Euler's totient function and thus, by Euler's extension of Fermat's little theorem, k must be a divisor of 108. Euler's Theorem guarantees that $2^{108} \equiv 1 \pmod{108}$. Suppose a smaller k also works. Every proper divisor of 108 divides either 36 or 54 (or both) so if any smaller k works, then one of $2^{36}, 2^{54}$ will be congruent to 1 mod 133. It will be convenient to have the powers of 2 mod 133; all the way to 2^32 (32 being the largest power of 2 < 54)

$$\begin{aligned}
2^0 &= 1, \quad 2^2 = 4, \quad 2^4 = 16, \\
2^8 &= 16^2 \equiv 123 \pmod{133}, \\
2^16 &\equiv 123^2 \equiv (-10)^2 = 100 \pmod{133}, \\
2^{32} &\equiv 100^2 \equiv (-33)^2 \equiv 1089 \equiv 25 \pmod{133}.
\end{aligned}$$

Since $36 = 32 + 4$ we get $2^{36} \equiv 25 \cdot 16 = 400 \equiv 1 \pmod{133}$. Is 36 the smallest? If there is a smaller k it has to divide 36; we can try 2^{18}. Since $18 = 16 + 2$ we have $2^{18} \equiv 100 \cdot 4 = 400 \equiv 1 \pmod{133}$. So $k = 18$ is possible, the only other case to rule out is $k = 9$. But $2^9 = 512 \equiv 113 \not\equiv 1 \pmod{133}$.

The answer is $\boxed{18}$

HQ9. Transcendental Matters

In 1873 Charles Hermite proved that e, the basis of natural logaritms, was transcendental. Using Hermite's ideas, and a bit more, Carl Louis Ferdinand von Lindemann proved in 1882, that π was transcendental, laying to rest 2000 years of people trying to square the circle. What is a transcendental number?

A. A very important number.
B. A number that is not the root of any algebraic equation with integer coefficients.
C. A number that is the solution of some algebraic equation with integer coefficients.
D. A number whose decimal expansion does not end nor is periodic.
E. An irrational number with an irrational natural logarithm.

Hermite (1822-1901)

Lindemann (1852-1939)

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MQ9. An Algebraic Number

The number $x = \sqrt{3} + \sqrt[3]{5}$ is algebraic, meaning it is not transcendental; it satisfies an equation of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$$

where $a_n,\ a_{n-1}, \ldots, a_0$ are integers, $n \geq 1$, $a_n \neq 0$. Find an equation of degree 6 satisfied by this x such that $a_6 > 0$ and the coefficients $a_6, a_5, a_4, a_3, a_2, a_1, a_0$ have no common divisor (other than 1). What is

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TIME’s UP!

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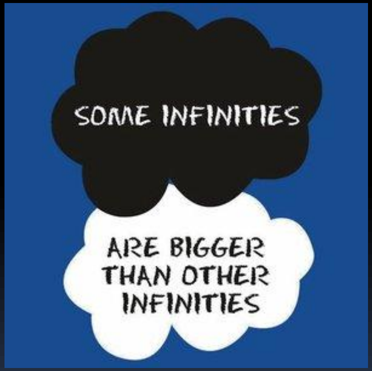
$$\begin{aligned}
(x-\sqrt{3})^3 &= (\sqrt[3]{5})^3,\\
x^3 - 3\sqrt{3}x^2 + 9x - 3\sqrt{3} &= 5,\\
x^3 + 9x - 5 &= 3\sqrt{3}(x^2+1)\\
(x^3+9x-5)^2 &= 27(x^2+1)^2\\
x^6 + 81x^2 + 25 + 18x^4 - 10x^3 - 90x &= 27x^4 + 54x^2 + 27\\
x^6 - 9x^4 - 10x^3 + 27x^2 - 90x - 2 &= 0
\end{aligned}$$

The answer is $1 + 9 + 10 + 27 + 90 + 2 = \boxed{139}$

HQ10. The Infinities Man



He saw that every infinity was part of a larger infinity, which was part of an even larger infinity, and so on; infinity swirling in infinity without end in sight. Did he go mad because of that? He was Georg Cantor (1845-1918). He assigned a cardinality to every infinite set; the smallest cardinality was that of sets that were in one-to-one correspondence with the natural numbers. His *continuum hypothesis* states

A. That all cardinalities are essentially the same.
B. That the set real numbers has a cardinality larger than the set of natural numbers.
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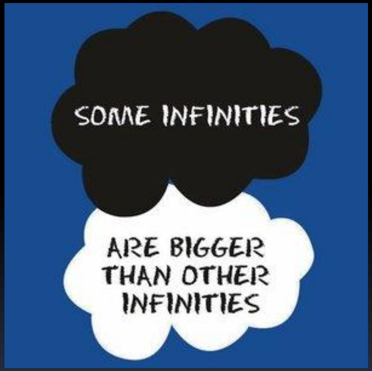

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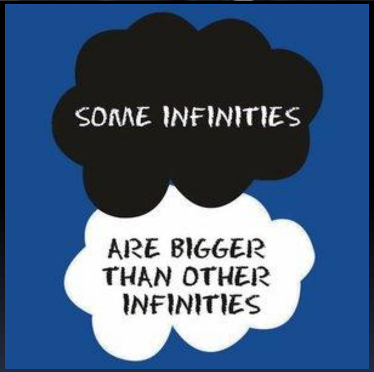

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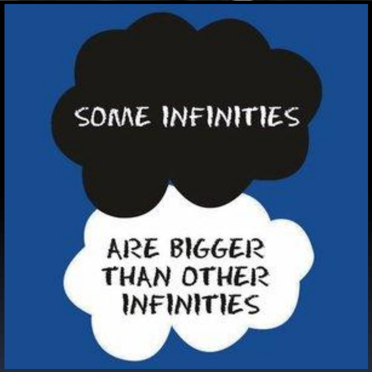

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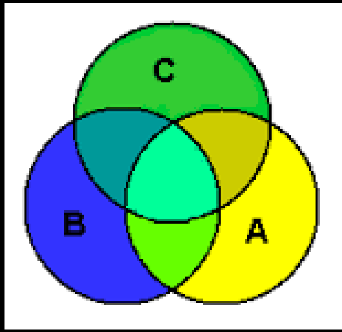
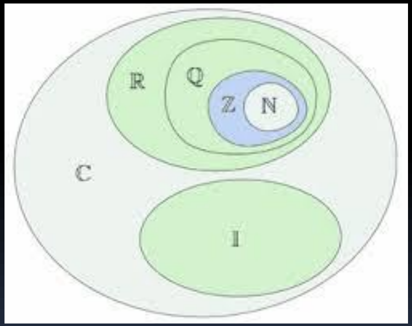
MQ10. Set Sensations



How many subsets of the set $\{1,2,3,4,\ldots,33,34\}$ have the property that the sum of the elements of the subset is greater than 297?



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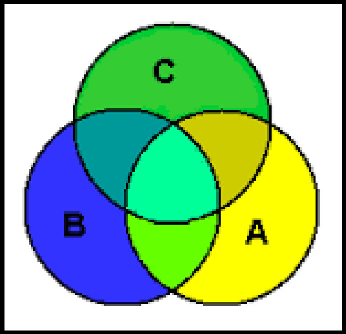

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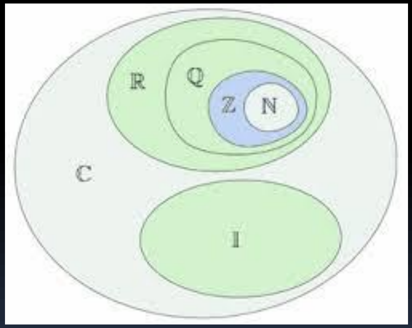

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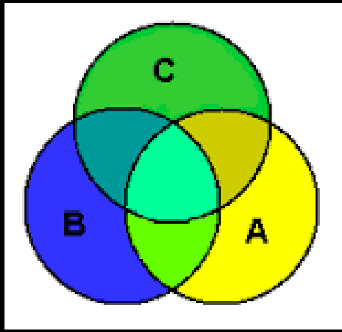

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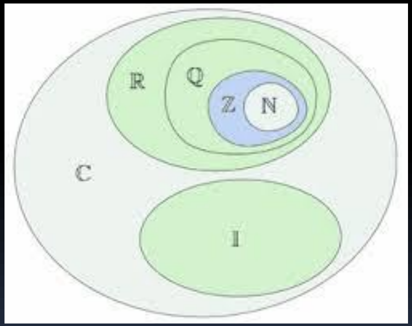

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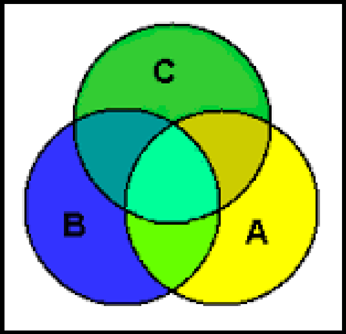

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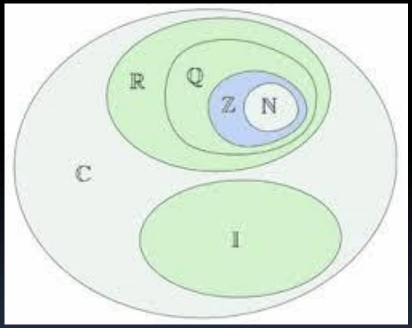

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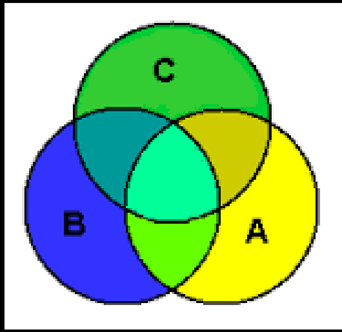

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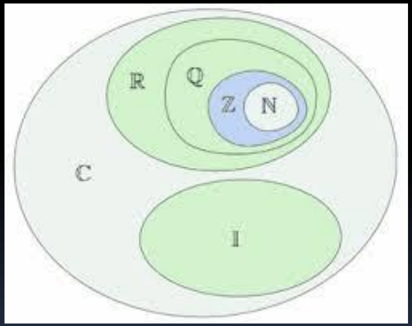
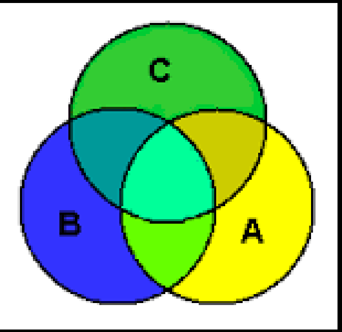


1

TIME's UP!

MQ10. Set Sensations



How many subsets of the set $\{1,2,3,4,\ldots,33,34\}$ have the property that the sum of the elements of the subset is greater than 297?

Notice that $1+2+3+\cdots+34=\frac{34\cdot35}{2}=595$ and 297=594/2. So if we partition the big set into any pair of sets, one of the pair must have a sum $\leq$ 297, the other one a sum >297. That means that there is a 1-1 correspondence between sets with sum less than or equal 297 and those with sum larger than 297; in other words, half the sets have a sum > 297.

2^{33} is the answer.

HQ11. A Great Algebraist

Being a woman, she had to fight anti-female bias from an early age. She was awarded a degree at the University of Göttingen, but then not allowed to teach. Things improved as her fame increased with publications of fundamental importance in abstract algebra. She finally got a position in Göttingen thanks to her expertise in invariant theory and its importance to relativity, only recently discovered. Being Jewish, she had to flee Germany when the Nazis came to power, and she became a professor at Bryn Mawr College. She was:

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20 seconds

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Time's Up!

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Time's Up!

MQ11. A Recursive Sequence

A sequence $a_0, a_1, a_2, a_3, a_4, \ldots$ is defined by
$a_0 = 3, a_1 = 4, a_2 = 8,$
And if $n \geq 3$, $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$.
Find a_{100}.
High exponents could be involved.

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High exponents could be involved.

The solution of a linear recurrence relation $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, where k is a fixed positive number and $c_1, \ldots, c_k$ are given constants, has the form $a_n = A_1 r_1^n + \cdots + A_k r_k^n$ for constants $A_1, \ldots, A_k$, where $r_1, \ldots, r_k$ are the roots of $r^n - c_1 r^{n-1} - \cdots - c_k = 0$, **if these k roots are distinct.** (If there are multiple roots, an adjustment is needed). In our case the equation to solve is $r^3 - 6r^2 + 11r - 6 = 0$. It is easy to guess the solutions $r = 1, 2, 3$ so the general solution has the form $a_n = A + 2^n B + 3^n C$. To find A, B, C we use the values provide for a_0, a_1, a_2, getting

$$A + B + C = 3, \quad A + 2B + 3C = 4, \quad A + 4B + 9C = 8.$$

Solving we get $A = 3, B = -1, C = 1$, Thus

$$a_{100} = \boxed{3 - 2^{100} + 3^{100}}$$