




An excursion through mathematics and its history

MATH DAY 2013—TEAM COMPETITION




A quick review of the rules

- History (or trivia) questions alternate with math questions
 - Math questions are numbered by MQ₁, MQ₂, etc. History questions by HQ₁, HQ₂, etc.
 - Math answers should be written on the appropriate sheet of the math answers booklet.
 - History questions are multiple choice, answered using the clicker.
 - Math questions are worth the number of points shown on the screen when the runner gets your answer sheet. That equals the number of minutes left to answer the question.
 - Have one team member control the clicker, another one the math answers booklet
- 



Rules--Continued

- All history/trivia questions are worth 1 point.
 - The team with the highest math score is considered first. Next comes the team with the highest overall score, from a school different from the school of the winning math team. Finally, the team with the highest history score from the remaining schools.
- 

HQ0-Warm Up, no points

- *Non Euclidean Geometry* is so called because:
 - A. It was invented by Non Euclid.
 - B. It negates Euclid's parallel postulate.
 - C. It negates all of Euclid's postulates.
 - D. Euclid did not care for it.
 - E. Nobody really knows why it is so called..

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Time's Up!


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Demonstrating the points system

- For math questions there will be a number in the lower right corner. It will change every minute. Here I am illustrating with numbers changing every 10 seconds. Try to imagine 10 seconds is a minute. The first number tells you the maximum number of points you can get for the question. Assume a question is on the screen.
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THE CHALLENGE BEGINS

VERY IMPORTANT!


Put away all electronic devices; including calculators.
Mechanical devices invented more than a hundred years ago,
are OK.





HQ1. Babylonians

One of the oldest of all known civilizations is that of the *Babylonians*, with capital in **Babylon**. Babylonians wrote mathematics (and other stuff) on:

- A. Papyrus.
 - B. Wax tablets.
 - C. Clay tablets.
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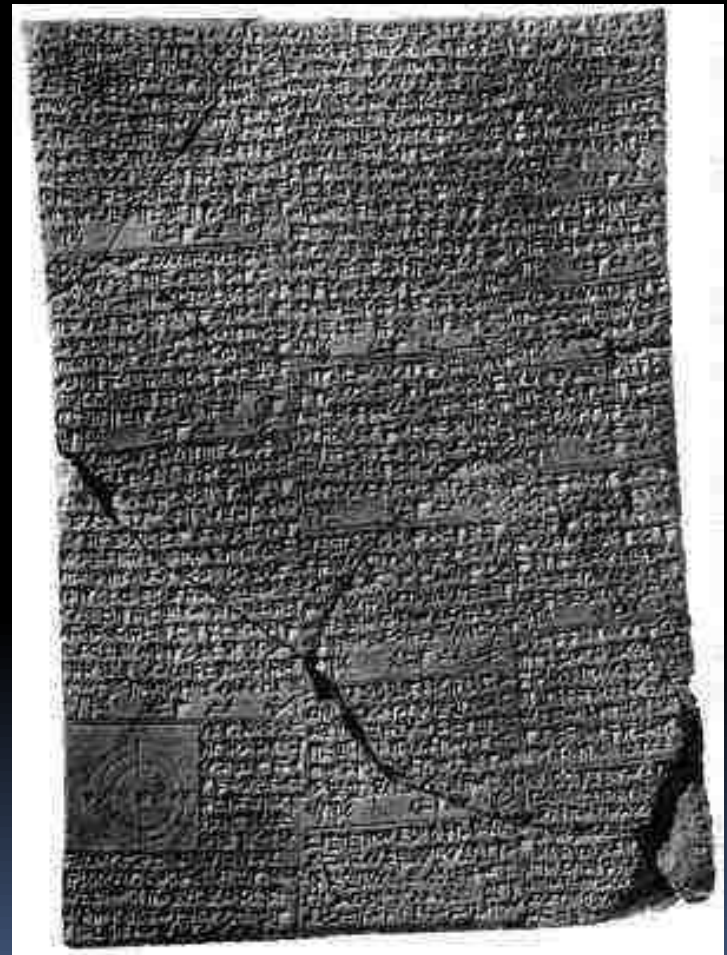
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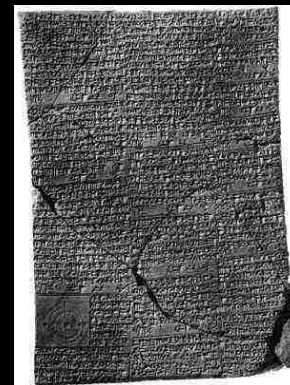
The Babylonians

The name Babylonians is given to the people living in the ancient Mesopotamia, the region between the rivers Tigris and Euphrates, modern day Iraq. They wrote on clay tablets like the one in the picture. The civilization lasted a millennium and a half, from about 2000 BCE to 500 BCE.



MQ1-Babylonian Tables

A Babylonian tablet has a table listing $n^3 + n^2$ for $n = 1$ to 30. Here are the first 10 entries of such a table.



n	$n^3 + n^2$
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100

Tables like these seem to have been used to solve cubic equations. The following problem appears in a tablet of 1800 BCE. The table on the left can help:

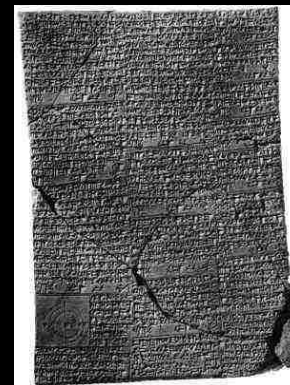
Solve for x, y, z

$$xyz + xy = \frac{7}{6}, \quad y = \frac{2x}{3}, \quad z = 12x$$

Write the value of $6(x + y + z)$ on the answer slip.

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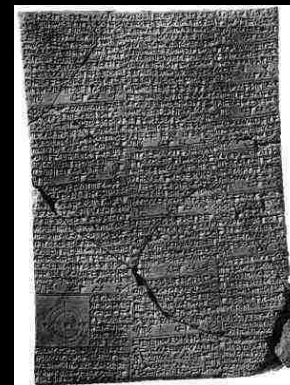
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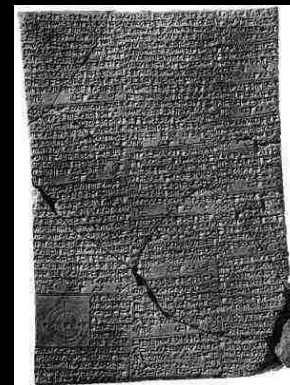
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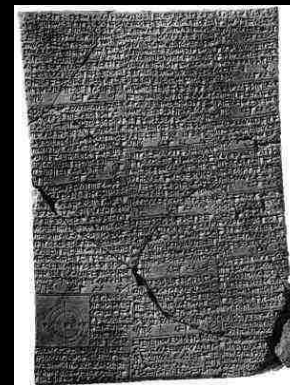
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TIME's  UP!

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Using the second and third equations to eliminate y, z from the first equation, we get

$$8x^3 + \frac{2x^2}{3} = \frac{7}{6}, \text{ or } 48x^3 + 4x^2 = 7.$$

One can probably guess a solution, but if we multiply the last equation by $36 = 6^2$ it becomes

$$(12x)^3 + (12x)^2 = 252.$$


From the table, $12x = 6$. Thus

$$\boxed{x = \frac{1}{2}, \quad y = \frac{1}{3}, \quad z = 6.}$$

Finally $6(x + y + z) = 6x + 6y + 6z = 41$



HQ2. The Greeks

- We will visit with the Greeks for a few questions. One of the first Greek mathematician was Thales of Miletus. The city of Miletus was in what is now which country?
 - A. Greece.
 - B. Macedonia.
 - C. Italy.
 - D. Turkey.
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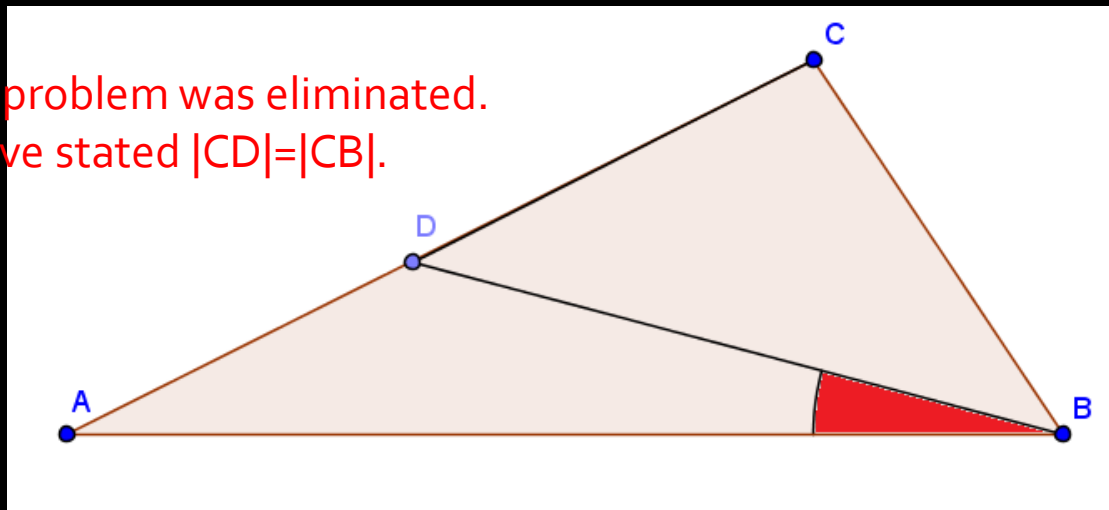
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Time's Up!

MQ2. What Thales Knew

NOTE: This problem was eliminated.
It should have stated $|CD|=|CB|$.



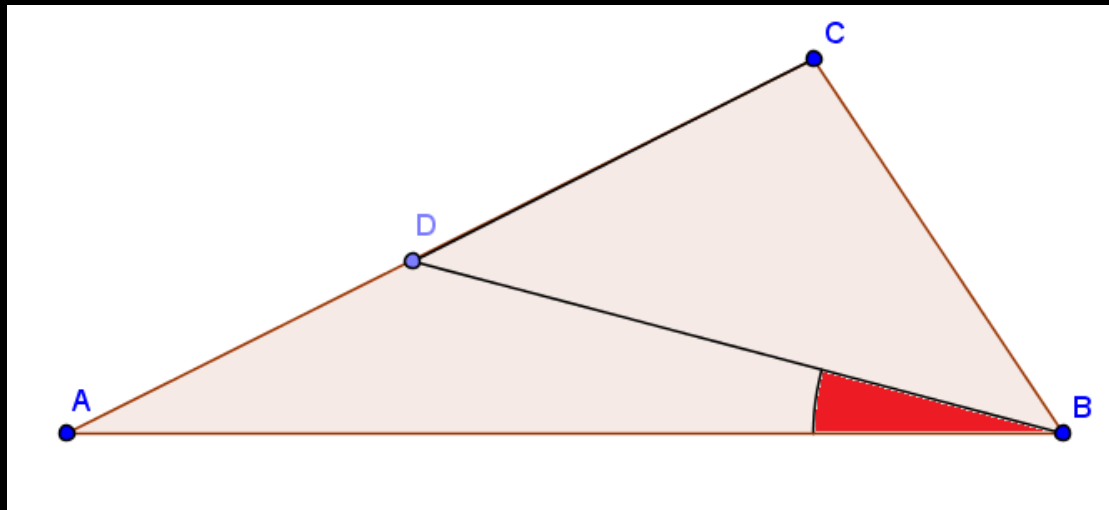
D is a point on side AC of triangle ABC .

The angle of the triangle at B minus the angle at $A = 30^\circ$.

That is, in symbols: $(\angle ABC) - (\angle BAC) = 30^\circ$.

What is $\angle ABD$ in degrees?

MQ2. What Thales Knew



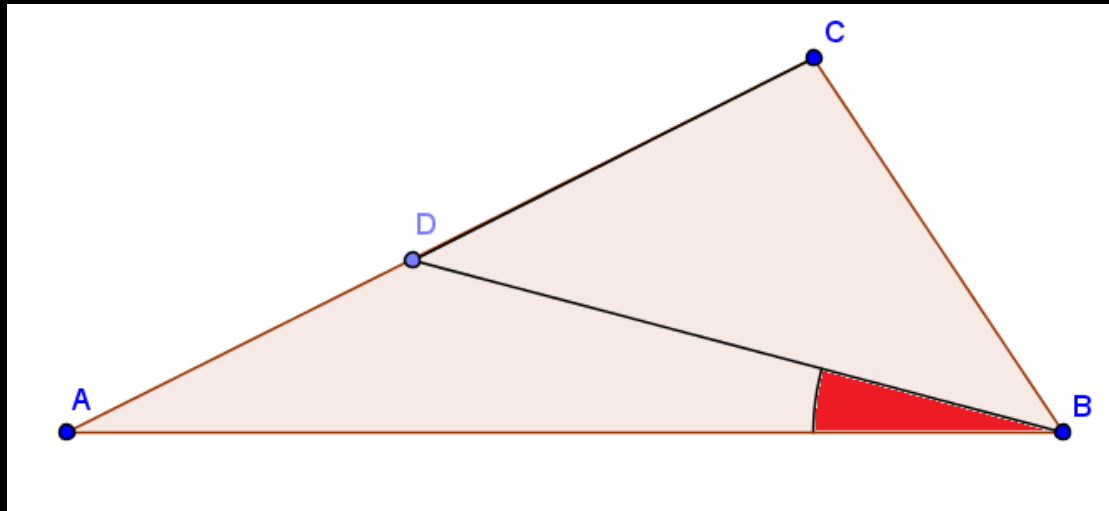
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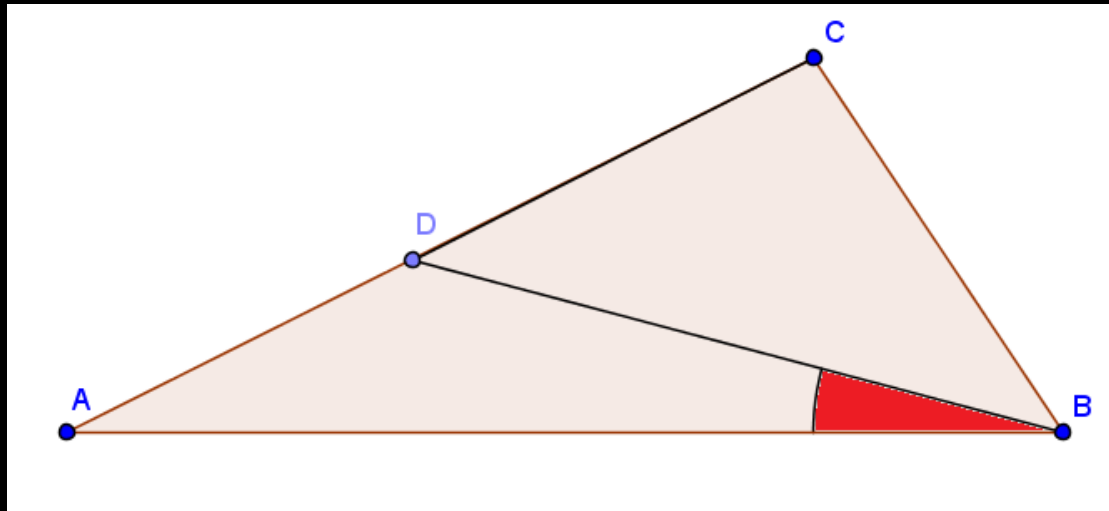
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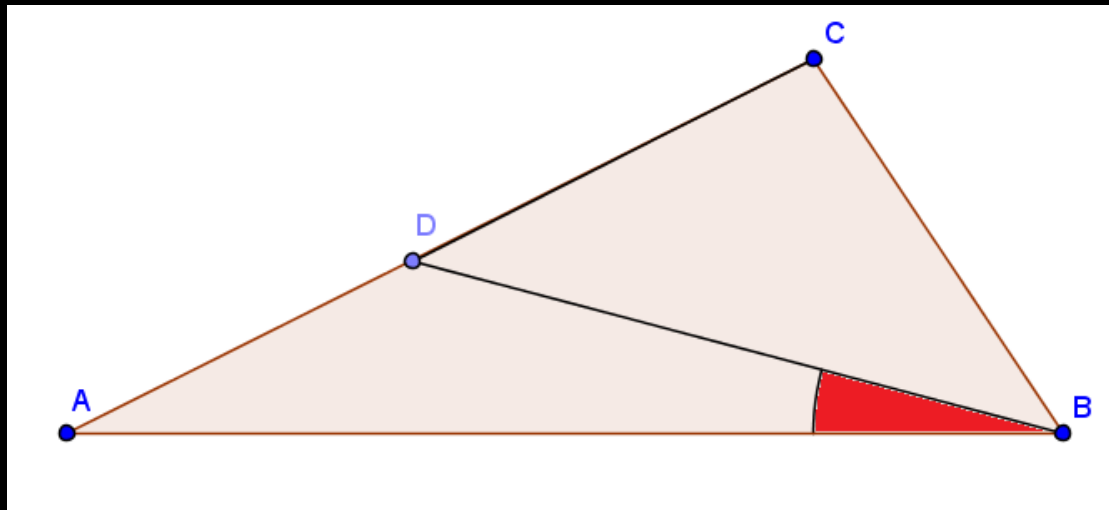
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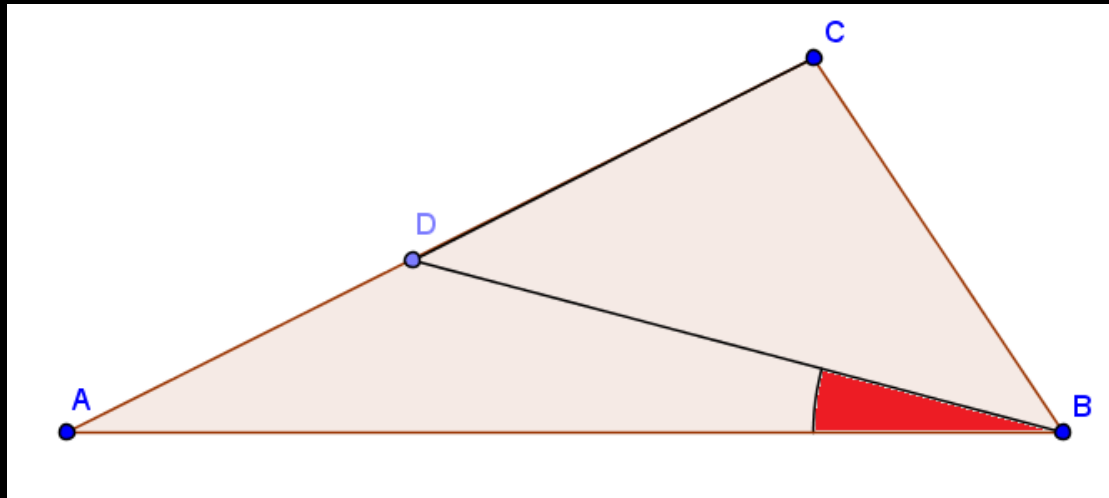
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$$\begin{aligned}\angle BDC &= \angle DBC \quad (\text{isosceles triangle}), \\ &= \angle ABC - \angle ABD, \quad \text{so } \angle BDC = \angle ABC - \angle ABD,\end{aligned}$$

$$\angle BAC + \angle ABD + \angle BDA = 180^\circ \quad (\text{sum of angles of a triangle}),$$

$$\angle BAC + \angle ABD = 180^\circ - \angle BDA = \angle BDC = \angle ABC - \angle ABD,$$

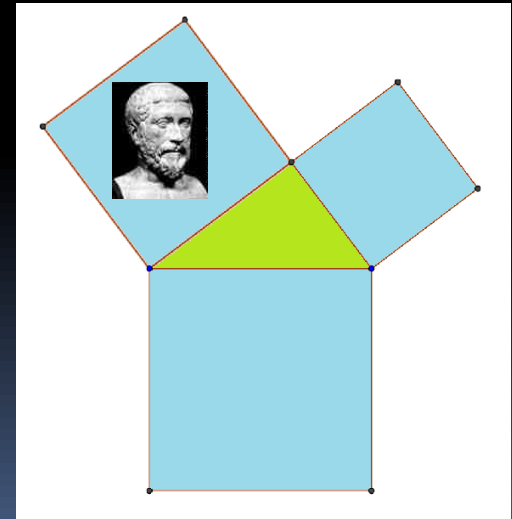
$$2\angle ABD = \angle ABC - \angle BAC = 30^\circ.$$

$$\boxed{\angle ABD = 15^\circ.}$$

HQ3. Pythagoras

After Thales, overlapping with him, came Pythagoras of Samos, who believed numbers ruled the universe. Pythagoras is supposed to have lived approximately how many years ago? (Select the closest answer).

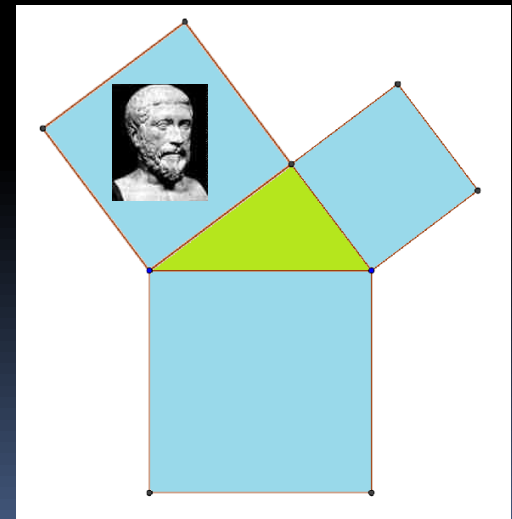
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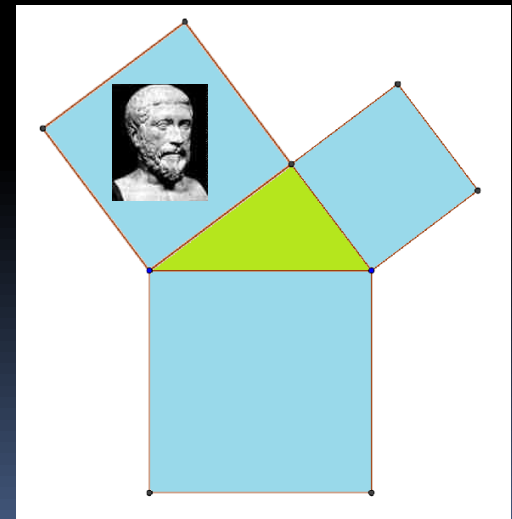


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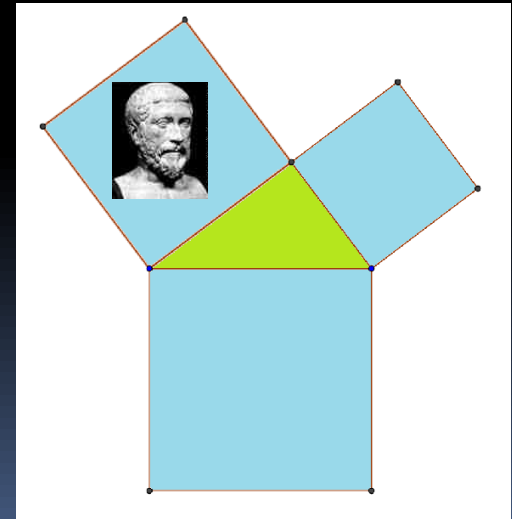


Times Up!

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MQ3. Primitive Pythagorean Triples



A **primitive** Pythagorean triple is a triple (a,b,c) of positive integers such that $a^2 + b^2 = c^2$ and such that a, b, c have no common divisor (except 1). For example $(3,4,5)$ is a Pythagorean triple. So is $(5,12,13)$. But $(9,12,15)$ is not, because 3 is a common divisor of 9, 12 and 15.

How many primitive Pythagorean triples (a,b,c) are there, with b even, and $c < 50$

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How many primitive Pythagorean triples (a,b,c) are there, with b even, and $c < 50$. The answer is **7**. They are: $(3,4,5)$, $(5,12,13)$, $(15,8,17)$, $(7,24,25)$, $(21,20,29)$, $(35,12,37)$. and $(9, 40, 44)$.

There is a formula for them; it helps to know it.

HQ4. Hippasus, why did you betray us?



One of the most important contributions of the Pythagoreans was the discovery of irrational numbers.

An irrational number is a number that

- A. Is not the quotient of two integers.
- B. Is the square root of an integer.
- C. Is a number with a non-terminating decimal expansion.
- D. Is a number that is between two consecutive rational numbers.
- E. Is a crazy number that thinks it is bigger than all other numbers.



Pythagoreans celebrating sunrise

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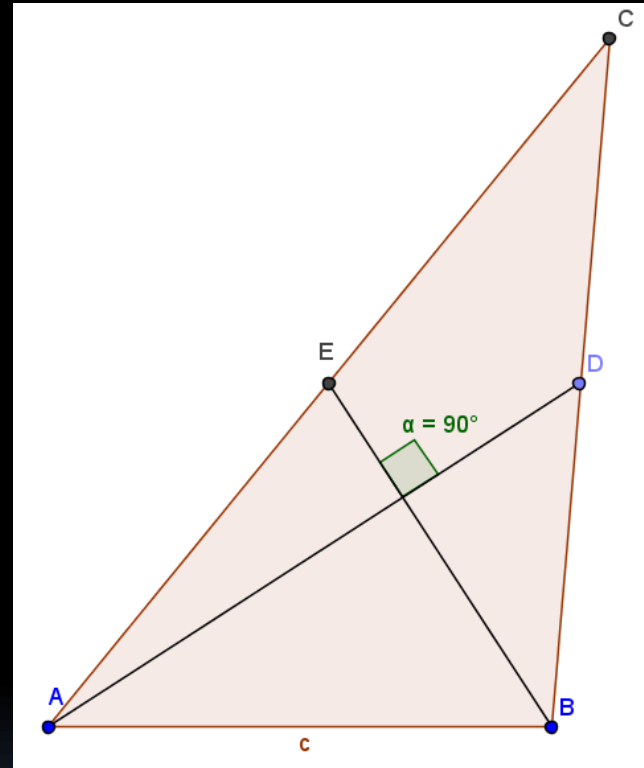
MQ4. Medians and Pythagoras

AD and BE are medians of triangle ABC, so $|AE| = |EC|$ and $|BD| = |DC|$.

If $|AC| = 9$ and $|BC| = 7$, what is

$$c^2 = |AB|^2 ?$$

A property of the medians makes this a not too terribly hard problem. And Pythagoras.



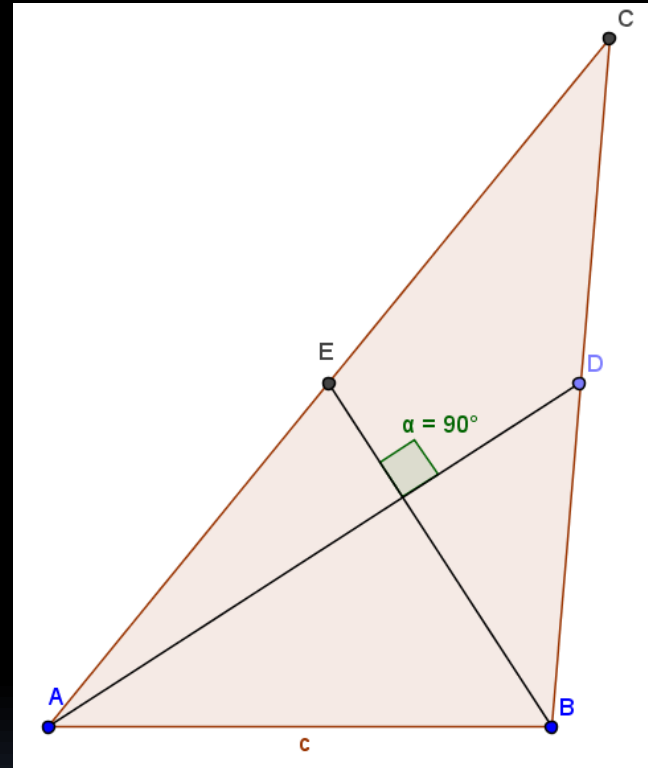
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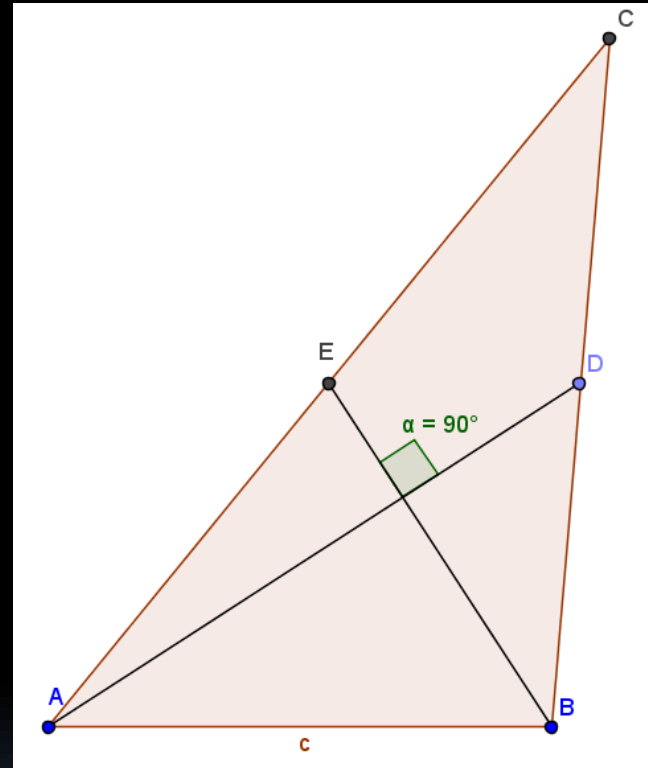
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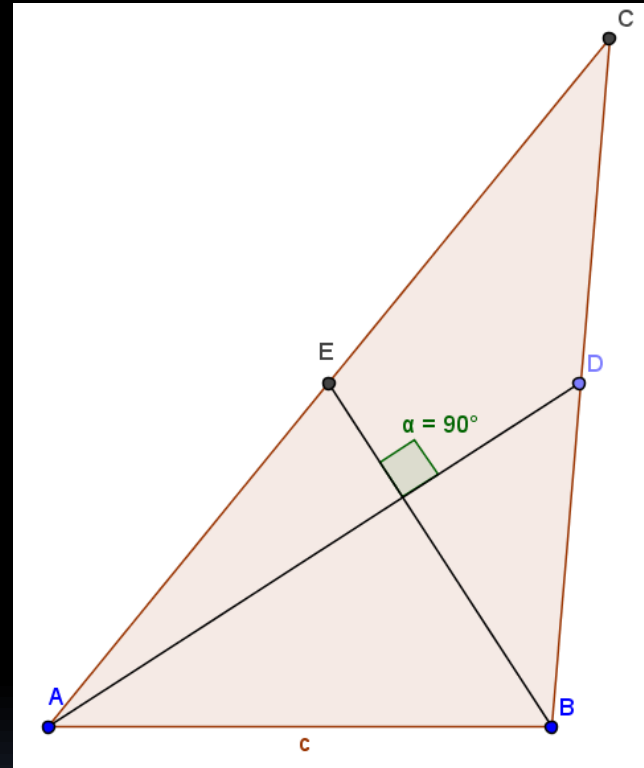
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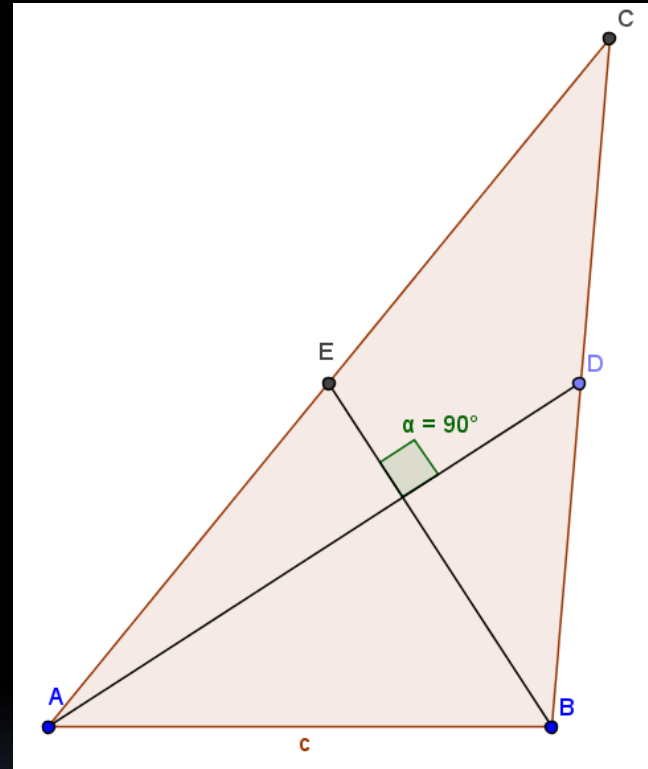
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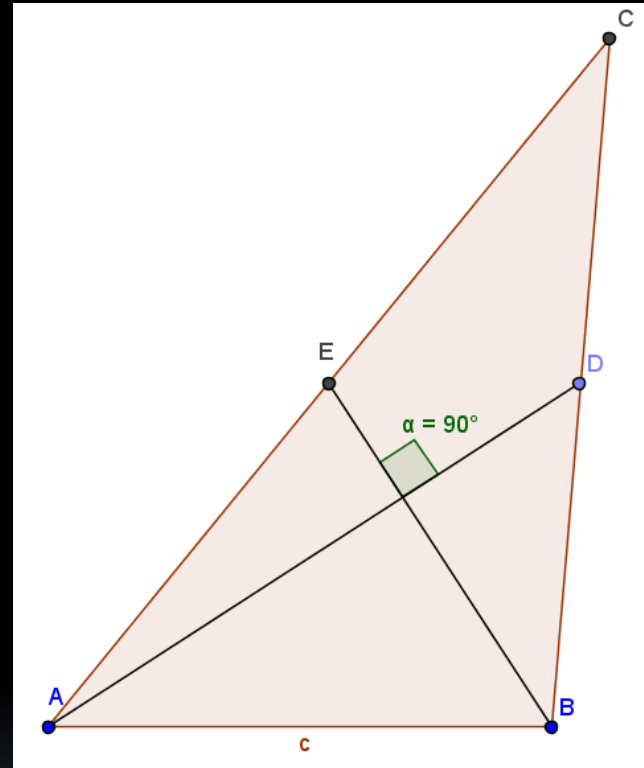
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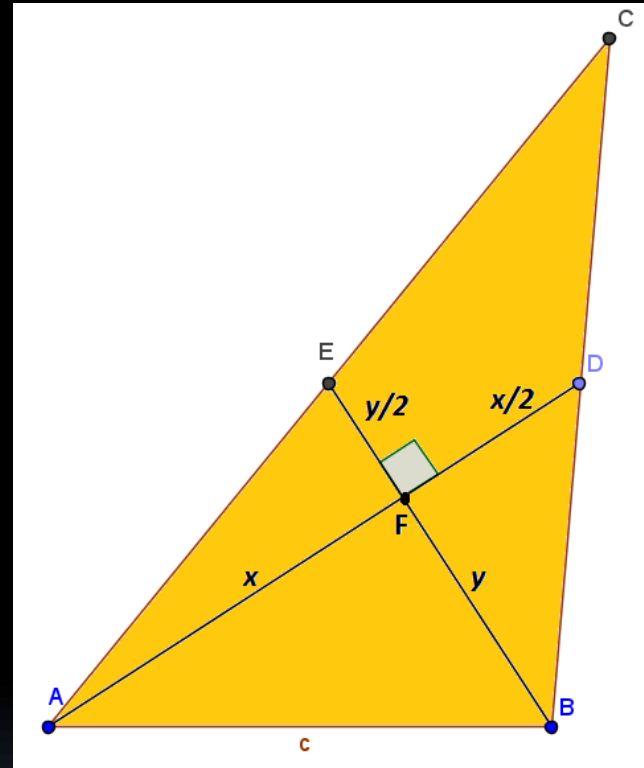
A property of the medians makes this a not too terribly hard problem. And Pythagoras.

The intersection F of the medians is the centroid of the triangle and $x = |AF| = 2|FD|$; $y = |BF| = 2|FE|$. By Pythagoras,

$$\begin{aligned}x^2 + \frac{y^2}{4} &= |AE|^2 = \frac{|AC|^2}{4} = \frac{81}{4}, \\ \frac{x^2}{4} + y^2 &= |BD|^2 = \frac{|BC|^2}{4} = \frac{49}{4}, \\ x^2 + y^2 &= c^2.\end{aligned}$$

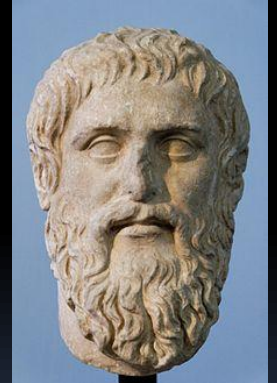
Solving the first two equations for x^2, y^2 , $x^2 = \frac{275}{15} = \frac{55}{3}$, $y^2 = \frac{115}{15} = \frac{23}{3}$, hence

$$c^2 = \frac{55}{3} + \frac{23}{3} = \frac{78}{3} = \boxed{26}.$$



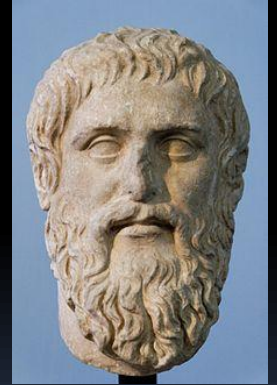
HQ5. Plato's ideal solids

- A Platonic solid is a polyhedron with the property that all faces are congruent, regular polygons, and the same number of faces meet at each vertex. How many such polyhedra are there?
 - A. 3
 - B. 4
 - C. 5
 - D. 6
 - E. An infinite number



HQ5. Plato's ideal solids

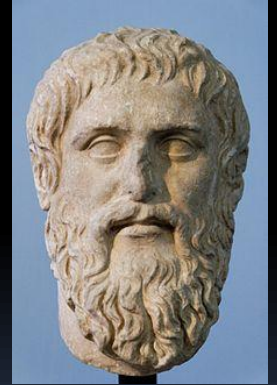
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20 seconds

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Time's Up!

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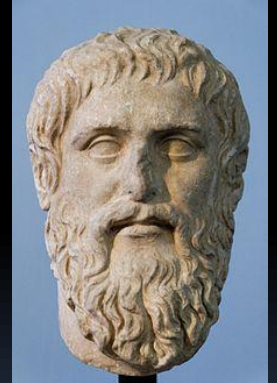
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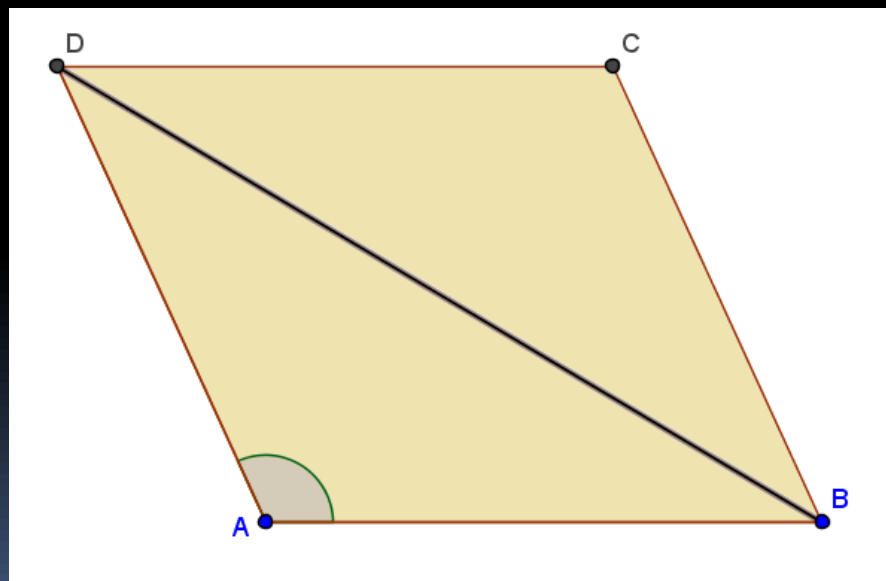


MQ6. Greek Heroes

All sides of parallelogram ABCD are of length 5: $|AB| = |BC| = |CD| = |DA| = 5$. If the area of the parallelogram equals 24, and the angle at A is obtuse, find $|BD|$.

Hints: It may simplify computations if you work with $x = |BD|/2$.

The title of this slide is also a hint.

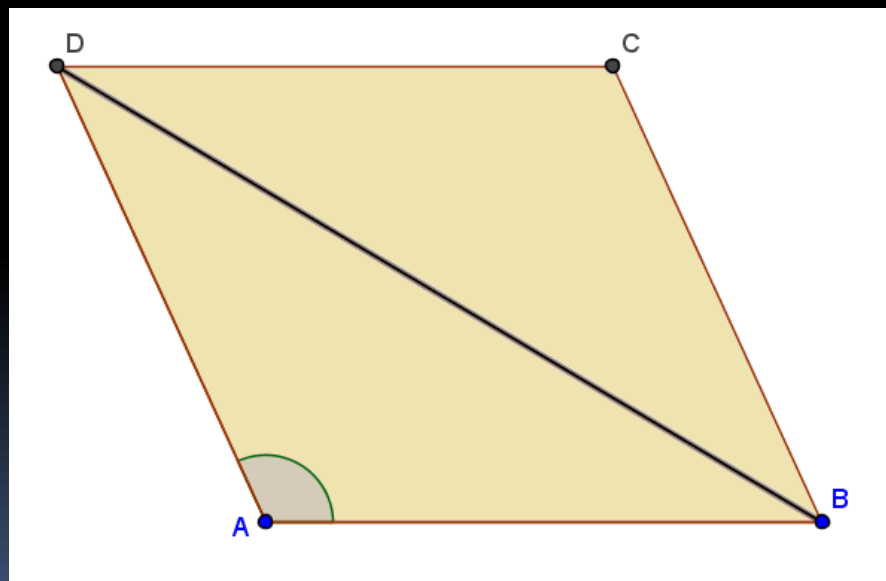


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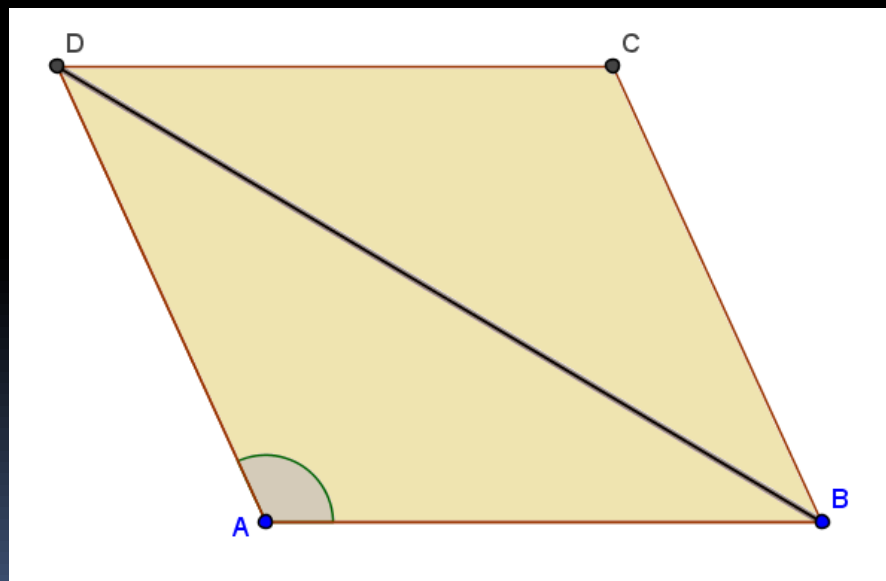


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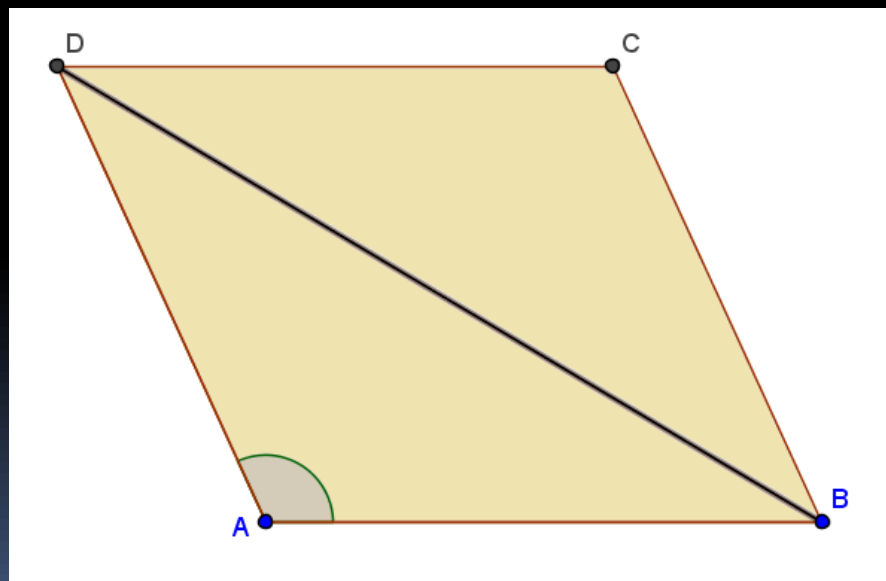


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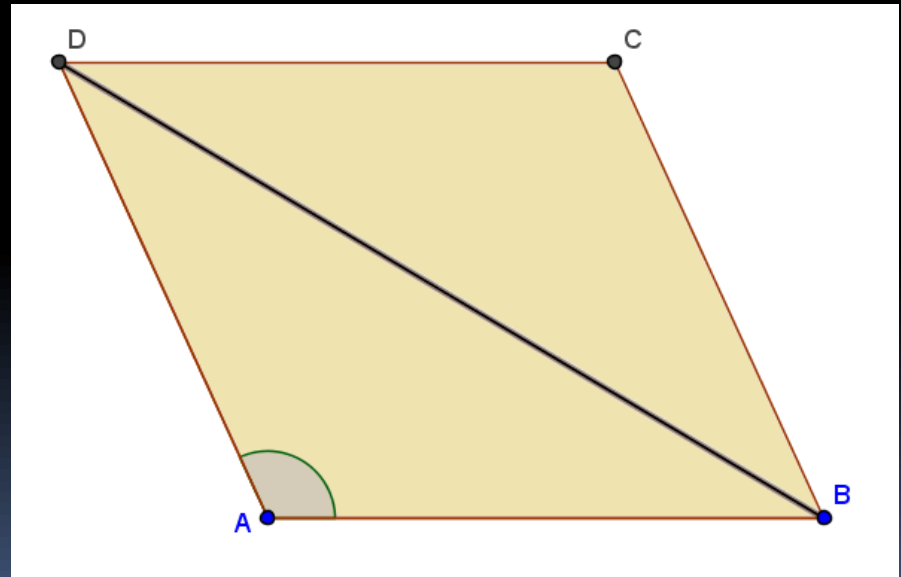
If the area of the parallelogram is 24, the area of triangle ABD is 12. By Heron's (a.k.a. Hero's) formula, setting $|BD| = 2x$, so $s = (5 + 5 + 2x)/2 = 5 + x$,

$$144 = 12^2 = s(s-5)(s-5)(s-2x) = (5+x)x^2(5-x) \\ = 25x^2 - x^4.$$

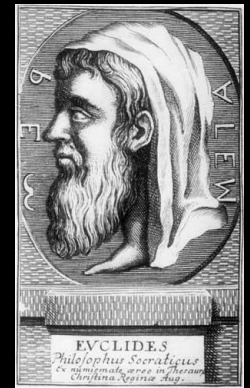
Rearranging, we get $x^4 - 25x^2 + 144 = 0$; a quadratic equation for x^2 , with solutions

$$x^2 = \frac{25 \pm \sqrt{625 - 576}}{2} = \frac{25 \pm 7}{2}.$$

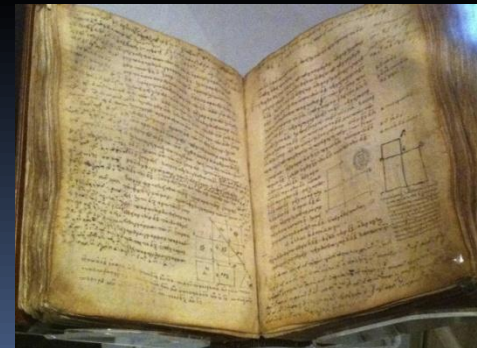
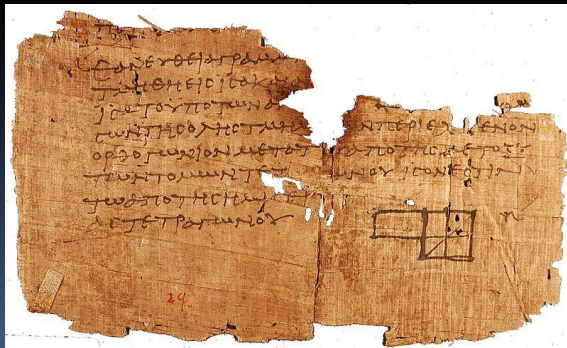
The two solutions are 9 and 16, thus $x = 3$ or 4 and $|BD| = 6$ or 8. Since the angle at A is obtuse, it is likely that the choice should be 8. We should select 8 as our answer, even if we don't know that for the angle to be obtuse it is a n.a.s.c $|BD| > \sqrt{2}|AB|$, so $|BD| > 5\sqrt{2} \approx 7$. The answer is $|BD| = 8$.



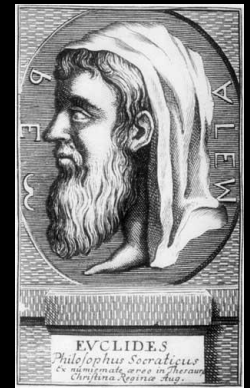
HQ7. One from Alexandria



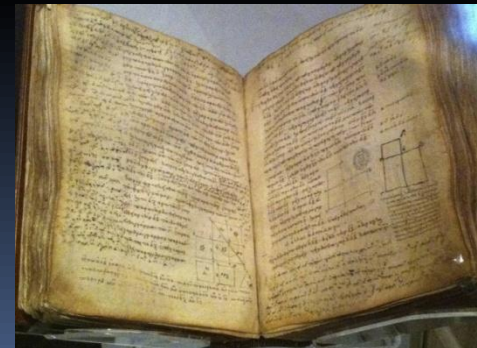
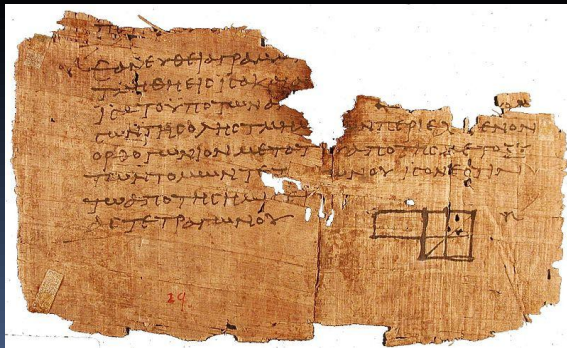
- Euclid's *Elements* is one of the most important, perhaps **the** most important scientific/mathematical book of antiquity. It is not only geometry; it contains all the mathematics known at the time, c. 300BCE



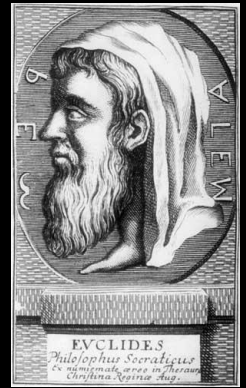
HQ7. One from Alexandria



- The 13 Volume Elements are not only geometry. One of the most famous results is the theorem proving the existence of an infinity of prime numbers.

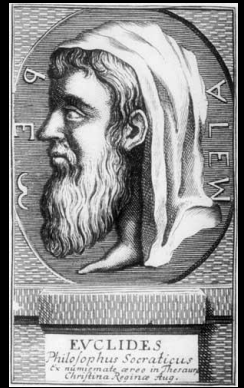


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- Another famous theorem is known as the *fundamental theorem of arithmetic*. It says (number=positive integer):
 - A. Every even number > 2 is the sum of two primes.
 - B. If a prime divides a product, it divides one of the factors.
 - C. Every number > 1 can be decomposed uniquely into a product of primes.
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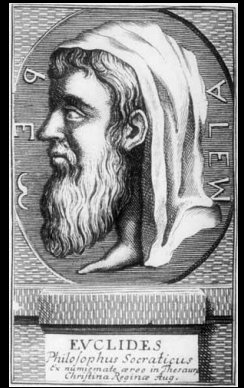
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Time's Up!

MQ7. A loosely related question.

- A store was unable to sell a supply of pens at \$0.50. Then it reduced the price and sold all for a total of \$31.93. All pens were sold for exactly the same price; the price was more than 1 cent per pen. How many pens were sold? (Same thing, how many pens were in the original supply?)

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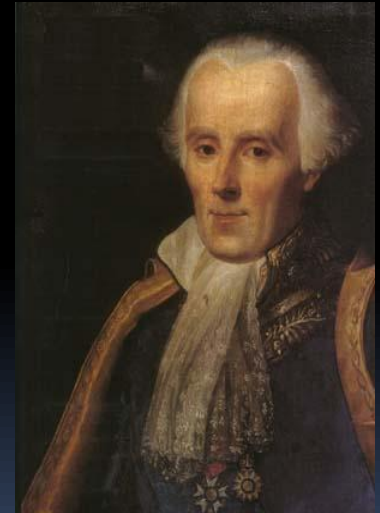
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- $3193 = 103 \times 31$, so **103** pens were sold at \$0.31 each.

HQ8. A Quote from Laplace

- *The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in _____. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated.*
- The blank refers to a country. It is
 - A. India.
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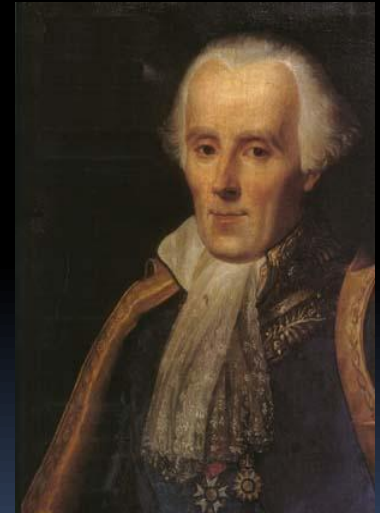


Pierre Simon,
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(1749-1827)

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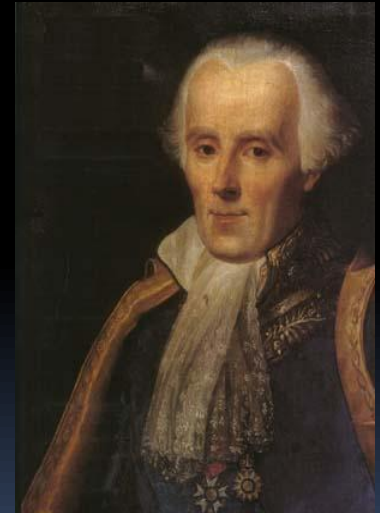


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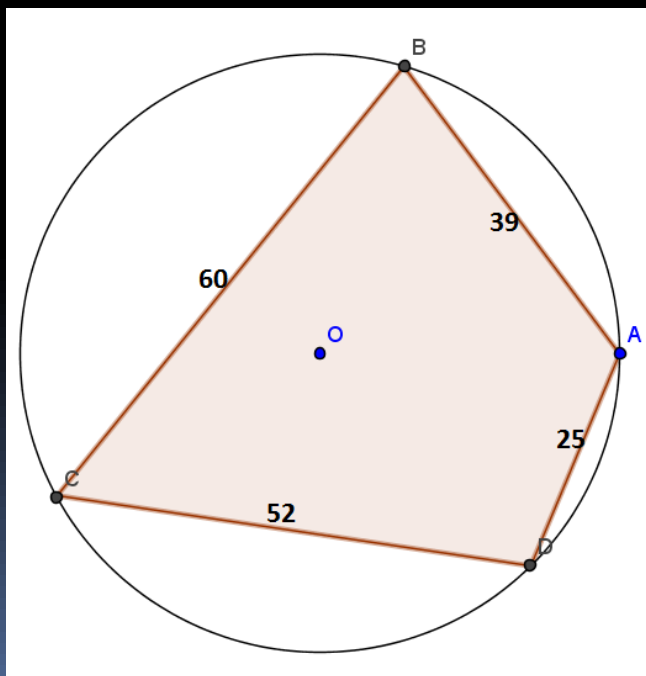
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MQ8. An Indian Trapeze Act

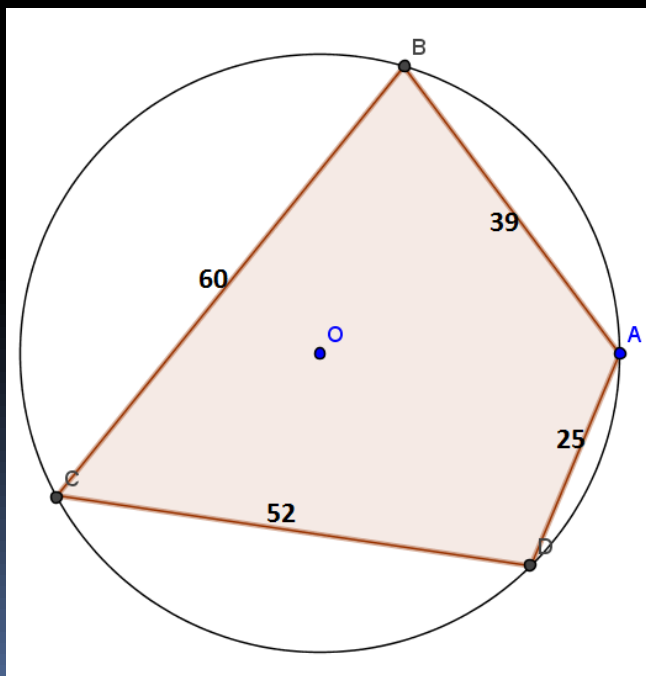
- A *Brahmagupta trapezium* is a quadrilateral formed from two Pythagorean triples (a,b,c) , (A,B,C) , of successive sides aC , cB , bC , cA . All are cyclical. The picture below shows one made from the triples $(3,4,5)$, $(5,12,13)$. Find its area.



Brahmagupta
ca. 620

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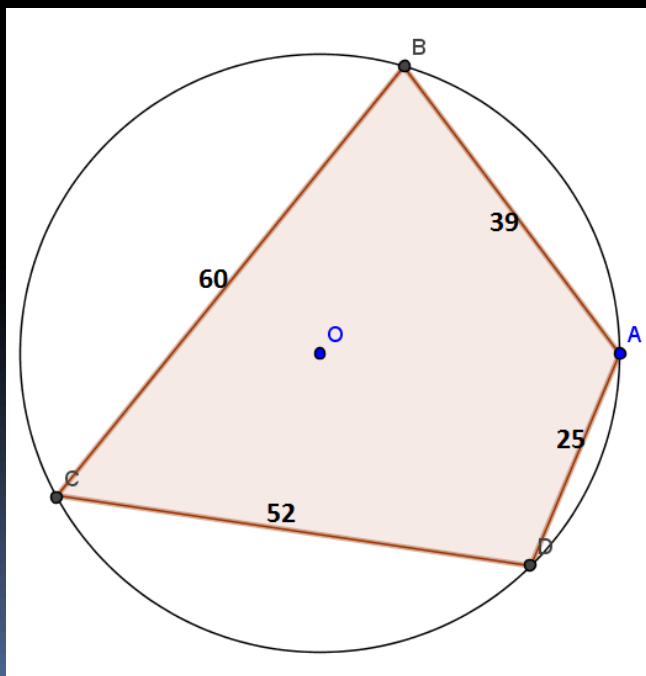
3



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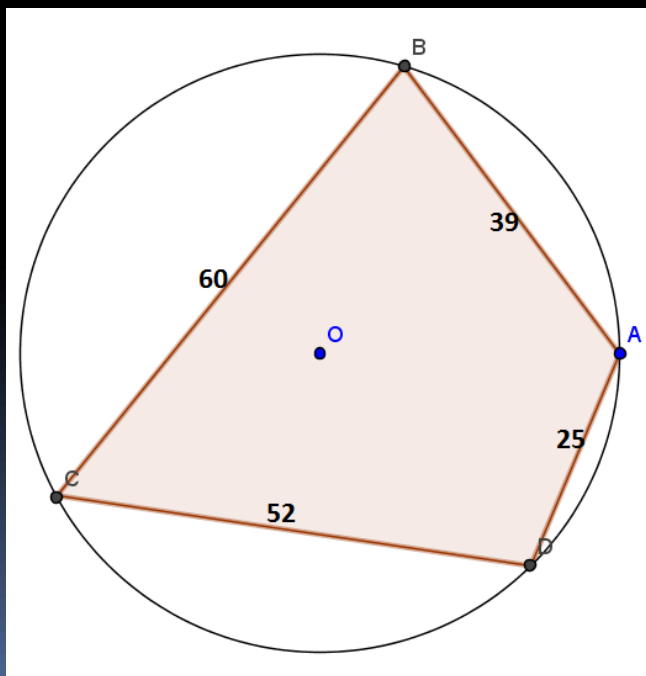
2



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1

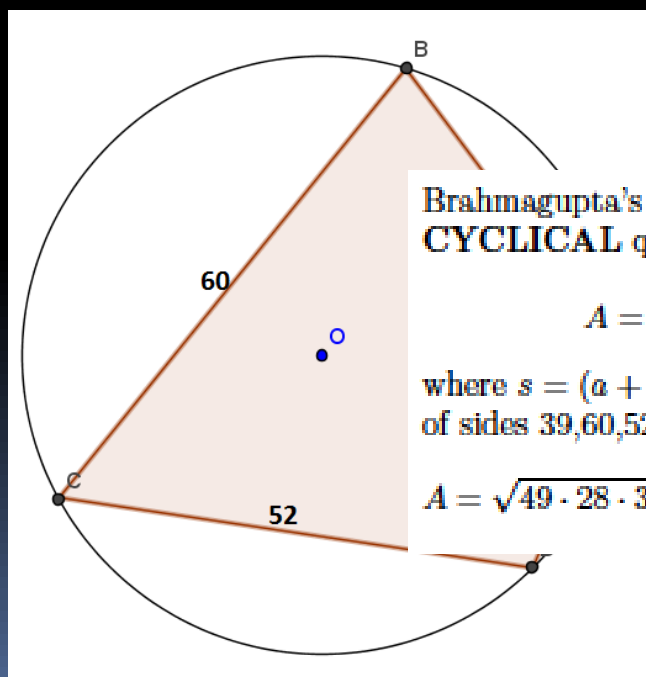


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TIME's  UP!

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Brahmagupta's formula for the area of a **CYCLICAL** quadrilateral of sides a, b, c, d is

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $s = (a + b + c + d)/2$. For a cyclical quadrilateral of sides 39,60,52,25, we get $s = 88$ thus

$$A = \sqrt{49 \cdot 28 \cdot 36 \cdot 63} = 7 \cdot 6 \sqrt{(4 \cdot 7)(9 \cdot 7)} = 7 \cdot 6 \cdot 2 \cdot 3 \cdot 7 = \boxed{1764}.$$

HQ9. Cubics

- The Babylonians, thousands of years ago, knew how to solve quadratic equations by the quadratic formula. More than two thousand years later people finally figured out how to similarly solve cubic equations, equations of the form $ax^3 + bx^2 + cx + d = 0$.

HQ9. Cubics

It was the 1500's, the Renaissance was beginning to get into full swing and a number of Italian mathematicians loosely associated with the University of Bologna did it. The formula is commonly known as:

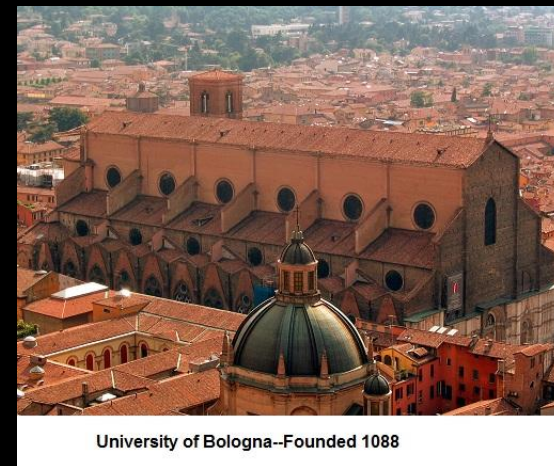
- A. The formula of Copernicus and Galileo.
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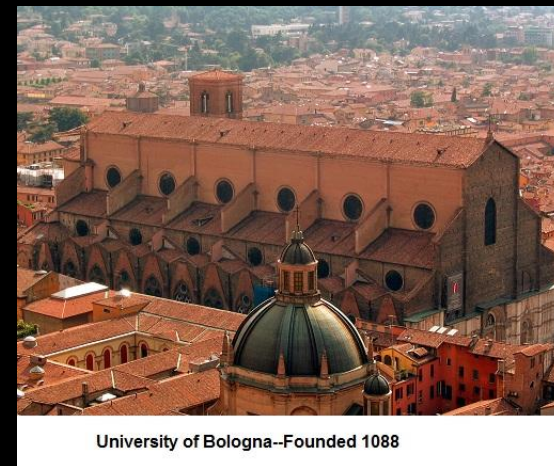


20 seconds

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Girolamo Cardano
(1501-1576)



Nicolo Tartaglia
(1499-1557)

MQ9. Nicolas Chuquet

- Two wine merchants enter Paris, one of them with 64 casks of wine, the other with 20. Since they do not have enough money to pay the custom duties, the first pays 5 casks of wine and 40 francs, and the second pays 2 casks of wine and receives 40 francs in change. What is the price of each cask of wine? Chuquet's *Triparty en la science des nombres* (1484)

Nicolas Chuquet (1445-1488) was a French mathematician, one of the first to use 0 as an exponent.



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Duty = 10fr., Cask price = 120 fr. So 120 is the answer.

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