# FAU Math Circle
**1/23/2016**

## Math Warm Up

- We start again in the island of knights, knaves, and normals. Knights always tell the truth, knaves always lie, and normals can go either way. We see three inhabitants A, B, C sitting in a row; A on the left, C on the right, B in the middle. We know that one of them is a knight, one is a knave, one is normal. You have to figure out who is what and you ask each one one question. You ask A, "What is the person sitting next to you," and A answers "a knight." You ask B "What are you," and B answers "I am as normal as can be." You ask C "What is the person sitting next to you," and C answers "a knave." What are A, B, C?

- Freddie says: "The day before yesterday I was 10, but next year I will turn 13." How can that be?

# Today's Problems

(10/24/15)
**Rules:**

1. Ms. Daisy is a great cat lover; she has 20 of them, adult cats and kittens. Each Sunday she gives a total of 92 cat treats to her cats; each grown cat gets 5 pieces and each kitten gets 3 pieces. How many of each kind does Ms. Daisy have?

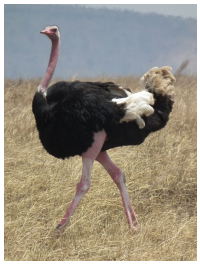
2. Old MacDonald has 32 animals on his farm, all pigs and hens. Together these animals have 104 legs. How many are pigs, how many are hens?


3. 

At the Magic Cookies and Candy Shop, a cookie costs twice as much as a piece of candy. Tim bought six cookies and three pieces of candy. Allie bought three cookies and six pieces of candy. Tim paid $1.80 more than Allie. What are the prices of cookies and candy.

4. 

Lion Country Safari has just received a new shipment of lions and ostriches. The new comers have a total of 33 heads and 110 legs. How many lions and how many ostriches were shipped?



5.

Jack's watch runs 1 second fast each hour, Jill's watch runs one and a half second slow. Suppose now both watches are showing the same time, and the time they show is the right time.

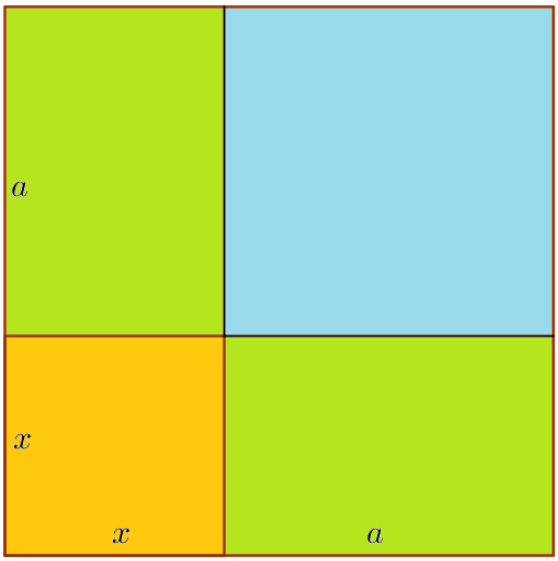
(a) When will both watches show the same time again?

(b) When will both show the same *correct* time again?

6. A useful formula is: If $a, x$ are numbers then

$$(x+a)^2 = x^2 + 2ax + a^2.$$

Explain what is the relation of the picture below to this formula.




7. The formula of the previous problem is very useful to solve equations of the form $x^2 + ax + b = 0$. For example, I'll use it to solve the following problem. *Four times a number and its square equal 21. Find the number.* If we call the number $x$, we see that

$$x^2 + 4x = 21$$

We make the left hand side look like the equation of the previous problem. The multiplier of $x$ is $2a$ there, here it is 4, so $2a = 4$, $a = 2$ and $a^2 = 4$. We add 4 to each side. The left hand side becomes $x^2 + 4x + 4 = (x+2)^2$; the right hand side becomes 25. So $(x+2)^2 = 25$ and a quick solution is $(x+2) = 5$ so $x = 3$.

(a) There is a second solution of $x^2 + 4x = 21$. Can you find it by this method? Did we miss it?

(b) Use the method to solve the following equations. Each one has two solutions. Find both.

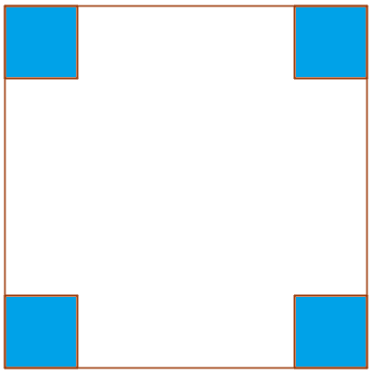
i. $x^2 + 8x = 20$.
ii. $x^2 + 9x = 10$.
iii. $x^2 + x = 1$.

8. The difference of two numbers is 40. The difference of their squares is 4800. What are they?

9. A certain positive number increased by its square equals 13,340. Find the number.

10. This was the last problem in last year's AMC 8 competition. Not many people got it right.

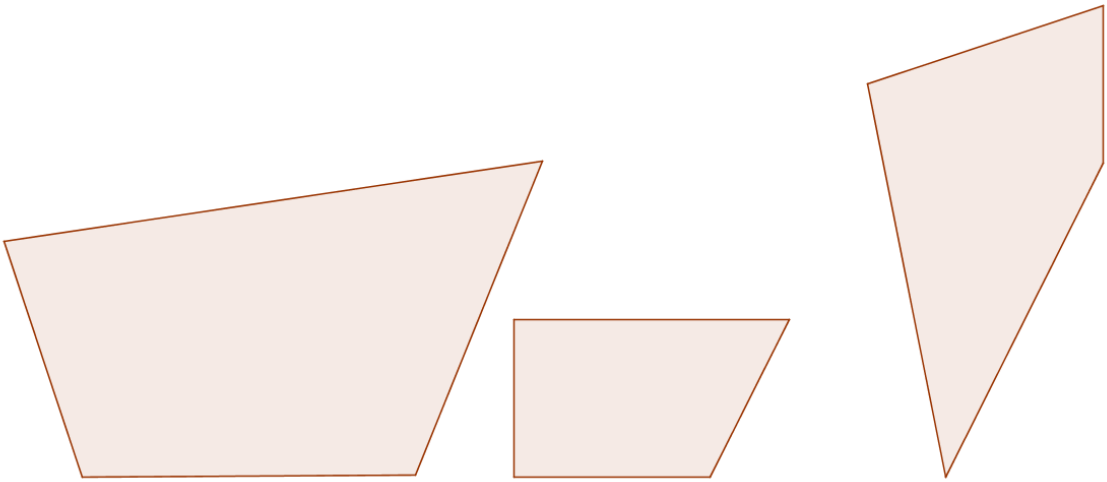
    One-inch squares are cut out from the corner of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?



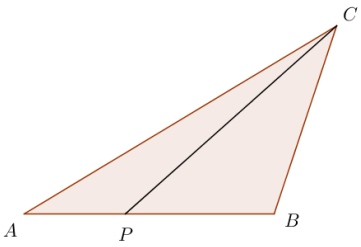
11. Here is a not too easy, but perhaps not incredibly hard, problem from last year's Mathworks competition.

    In a quadrilateral ABCD, the diagonals AC and BD intersect at point O. Suppose that triangles ABO, BCO, CDO have areas of 30, 40, 20 square units, respectively. Find the area of the triangle DAO.

    Here are a few quadrilaterals for you to work on but, of course, you can draw your own.



**Possible Hint:** Given a triangle $ABC$ and a point $P$ on one of its sides, say on side $AB$, is there any relation between the areas of triangles $APC$, $PCB$, and segments $AP, PB$? **Justify whatever relation you find!**




**Credits**

- The first warm up question is adapted from a puzzle book by Martin Gardner; other questions come from Anna Burago's *Mathematical Circles Diaries, Year 1.*
- The picture of the lion is *Lion Pose* by Michael Day.