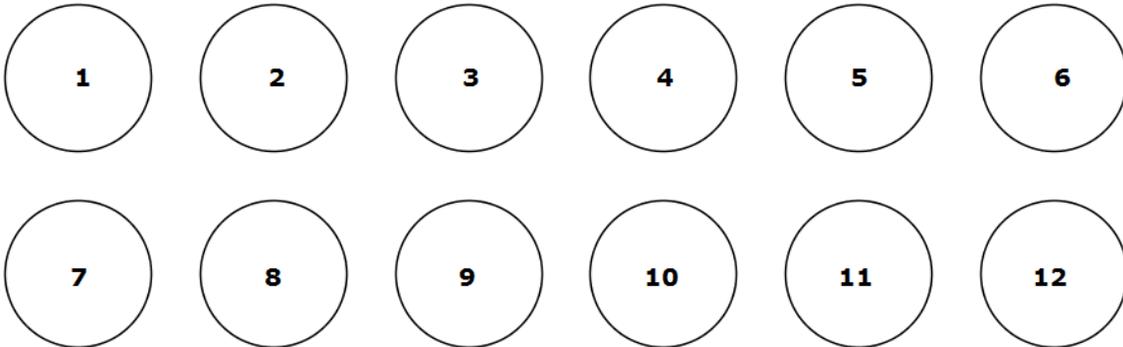


FAU Math Circle
12/5/2015

THE END OF YEAR 2015 COMPETITION

RULES

- Some of these problems are easy, some are hard. It is quite likely that nobody will be able to do all of them, so do as much as you can.
- Once you finish a problem go and show your answer to one of the organizers. Be prepared to have to explain how you arrived at the solution. If the organizer verifies that you have the correct solution, he or she will stick a signed star in the appropriate circle at the bottom of this page.
- You can ask one of the organizers for help. They will not solve the problem for you, but give you some hints on how to proceed.
- The competition will be over at 3:40. At that time stars will be given for all correctly solved problems not yet starred.
- Whoever collected the largest number of stars is the winner.



1. Erika promised to sell an average of 20 boxes of girl scout cookies per week over a period of six weeks. In the first five weeks she sold 15, 25, 18, 19, and 20 boxes. How many boxes does she have to sell in the sixth week to keep her promise? (AMC8)

2. Find the area of the figure in the following diagram, if the areas of the squares is 1.

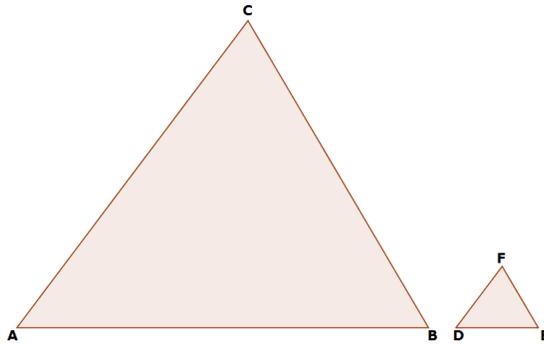


3. Here is a puzzle that is over 1700 years old:

Demochares (just call him D) has lived a fourth of his life as a boy, a fifth as a youth, a third as a man, and 13 years as an old man (people got old at a young age in those far away days). How old is he?

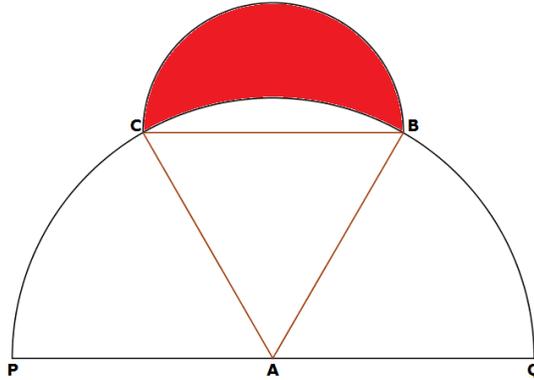
4. In an election 5219 votes were cast for four candidates: Amy, Beth, Meg and Jo. Amy won getting 22 more votes than Beth, 30 more votes than Meg, and 73 more votes than Jo. How many votes did each candidate get?(Sam Lloyd Puzzles)

6. Triangle ABC is similar to triangle DEF . If $|DE| = 15$, $|DF| = 14$, and $|EF| = 13$, and the area of triangle ABC is 24,276 square feet, what are the lengths of AB , AC , and BC ? Can our friend Heron be of any help? (Mathematical Wrinkles)



NOTE: The figures are **NOT** drawn to scale. If drawn accurately, $\triangle ABC$ would have to be much, much larger. Or $\triangle DEF$ much, much smaller.

7. The triangle ABC is an equilateral triangle having its vertex A at the midpoint of the diameter PQ of a halfcircle; its vertices B, C on the halfcircle. Another halfcircle is drawn with diameter BC . Find the area of the moonlike (crescent) region, called a *lune*, which is French for *moon*, shaded in red, knowing that $|PQ| = 4$ units. **Warning:** π and at least one square root will appear in the answer.

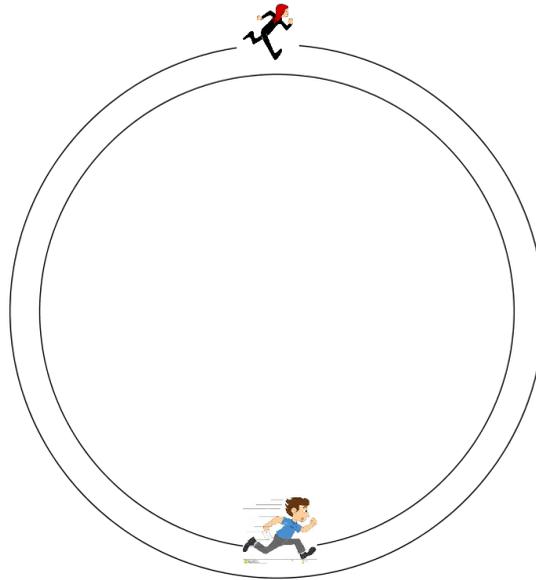


8. A positive integer plus its cube equals 592788. What is the integer? It may help to know that

$$592788 = 2 \times 2 \times 3 \times 7 \times 7057.$$

All of these factors are prime factors. (Mathematical Wrinkles, maybe)

9. Jack and Jill run in opposite directions on a circular track, starting at diametrically opposite points, at the same time. They first meet after Jill has run 300 feet. They next meet after Jack has run 450 feet past their first meeting point. Both run at a constant speed. What is the length of the track in feet?

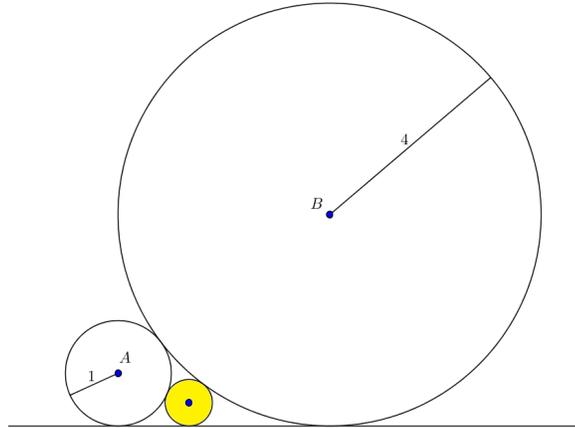


10.

A logging company wants to chop down a forest that is 99% pine trees. The Forest Service objects and the logging company, whose CEO is a devious mathematician, suggests that it will only cut pine trees and, when done, the forest will still be 98% pine trees. What percentage of the forest will be chopped down? (A Moscow Math Circle)



11. Three circles are tangent to a horizontal line and to each other, as shown in the figure below. The circle centered at A has a radius of 1; the circle centered at B has a radius of 4. What is the radius of the third yellow circle? The Theorem of Pythagoras could come in handy.



12. A circle of radius 7 inches is inscribed in a triangle of perimeter P inches and area A square inches. What is the ratio P/A ? The answer should be in one of the following forms:

- An integer.
- an expression of the form $\frac{a\sqrt{b}}{c}$ where a, b, c are integers.
- A fraction $\frac{a}{b}$, where a, b are integers with no common divisor other than 1.

