

Today's Problems

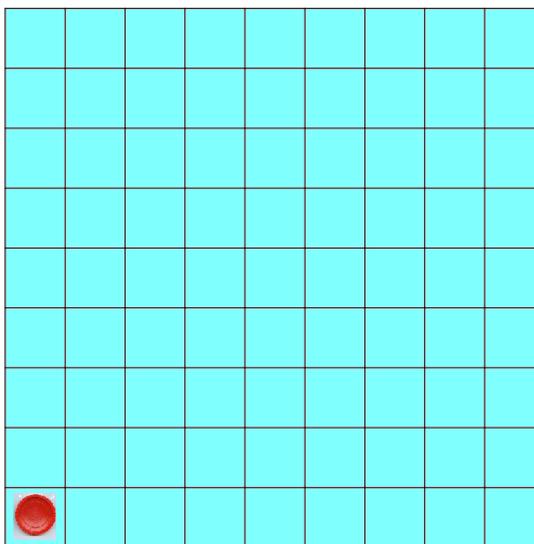
(10/24/15)

Rules:

- Work on these problems in any order. You will have until about 3:30 for this activity.
- Work alone or in groups.
- **This is NOT an exam. If you have questions, need hints, just ask one of the organizers.** We want to challenge you, not frustrate you.
- Feel free to get up, walk around the room, write on the white boards with the provided markers.
- At 3:30PM, more or less I will ask for solutions, and we will discuss the solutions. Students or groups who have found solutions, time permitting, can present them on one of the white boards.

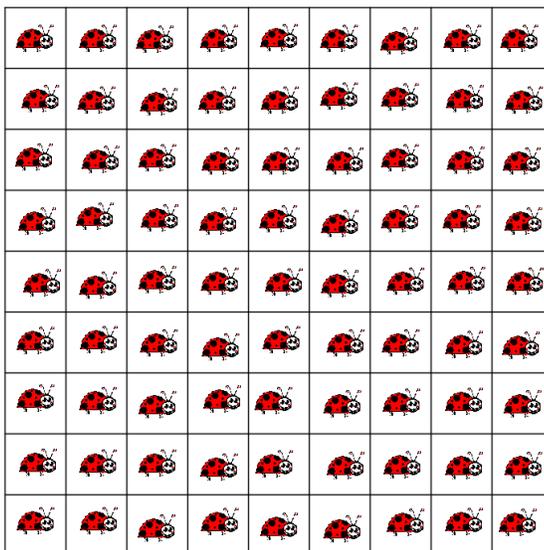
1. **(a)** There are three apples on a table. The first weighs 200 grams, the second 300 grams, and the third weighs 400 grams. Gabriela and Sam each take an apple and start to eat at equal rates. Whoever finishes an apple first takes the last one. If each wants to eat as much as possible, which apple should Gabriela take first? **(b)** What if there is a fourth 450 grams apple on the table?
2. Twenty points are placed on a sheet of paper and a line is drawn through every two points. What is the least and what is the greatest number of lines that could be so formed?
3. A chunk of consecutive numbered pages has fallen out of a folder. The first page of the chunk has number 463, and the last has the same digits but in a different order. How many sheets of paper were dropped? (Each sheet is two pages with consecutive numbers.)
4. Mike rips a piece of newspaper into 8 pieces. He rips one of the resulting pieces into 8 pieces. Can he rip the paper into 2015 pieces? If he waits until next year, can he rip it into 2016 pieces? If he waits two years, can he rip it into 2017 pieces? (Assume the original sheet of newspaper is humongous)
- 5.

A marker has been placed in the lower left corner of a 9×9 board. Two players take turns choosing a direction, right or up, and the number of spaces to move in that direction. The winner is the last one to make a move. Which of the players, first or second to move, can always win, and what is the strategy to do so.



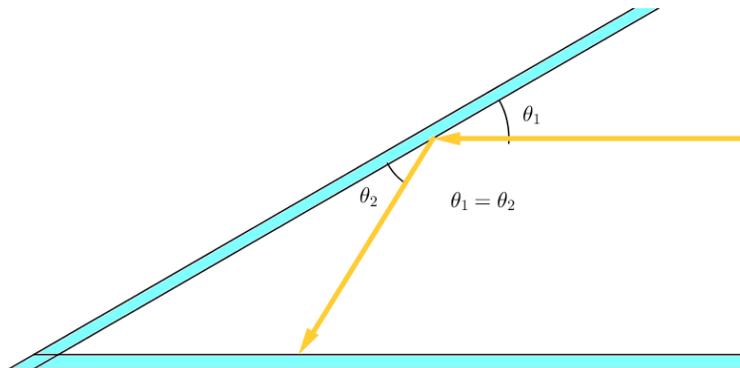
6.

Each square of a 9×9 board has a bug sitting on it. On a signal, each bug crawls onto one of the squares that shares a side with the one the bug was on. **(a)** Prove that one of the squares is now empty (that is, there is at least one square now with two or more bugs on it). **(b)** Can the bugs move so that there will be exactly one empty square?



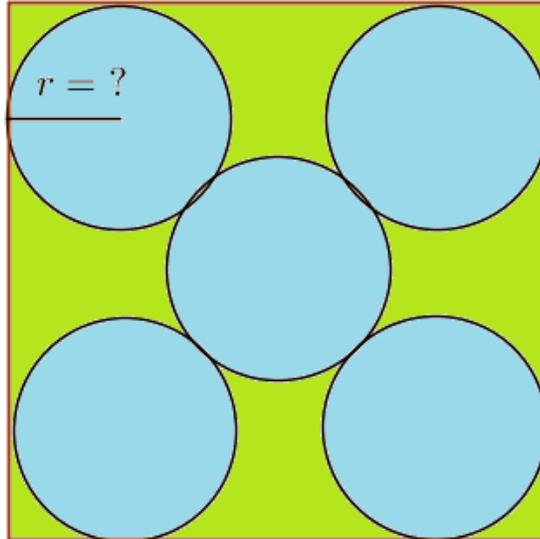
7. Sam and Alex live in the same apartment building and leave for school at the same time. Each of Sam's steps is 10% longer than Alex's, but Sam takes 10% fewer steps per minute than Alex. Who will get to school first?
8. (a) Two mirrors form a 30° angle. A light beam enters this angle parallel to one of the sides and is reflected from the sides according to the usual law that the angle of incidence is equal to the angle of reflection. Prove that the beam will eventually leave the angle. How many times will it reflect off the mirrors before leaving? (b) What if the angle between the mirrors is 20° ? (c) What if it is 50° ?

The picture shows a light ray entering the angle and the first reflection.



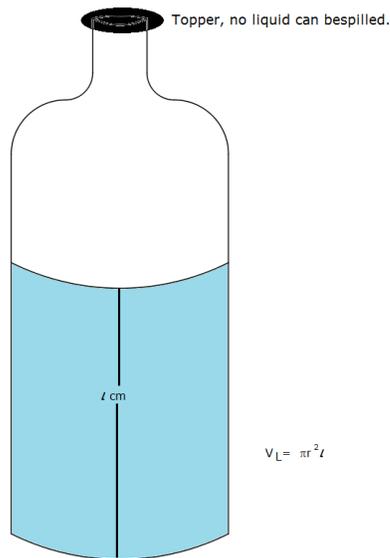
- 9.

The picture on the right shows a square and five circles of the same size inside the square. The four corner circles are tangent to the middle circle and to the sides of the square. If the side of the square has length L , what is the length of the common radius of the circles? (Spoiler alert: A correct answer involves a square root).



10.

A bottle with circular bottom is partially filled with liquid. There is a stopper at the top so no liquid can be spilled. You want to measure its volume but you only have a ruler. How can you measure its volume? It helps to know that the volume of a circular cylinder having a base of radius r and height h is $V = \pi hr^2$. Suppose the center of the base of the bottle is marked, so measuring the radius is easy.



11. Solve the following cryptic multiplication. Each asterisk stands for a digit and each digit is a prime (2, 3, 5, or 7).

$$\begin{array}{r}
 * * * \\
 \times * * \\
 \hline
 * * * * \\
 * * * * \\
 \hline
 * * * * *
 \end{array}$$

Is there more than one solution?