FAU Math Circle
10/3/2015

Math Warm Up

- The National Mathematics Salute!!! (Ana)

- What is the correct way of saying it: 5 and 6 are 12 or 5 and 6 is 12?

- For the next three questions we turn to the island of Knight and Knaves. This island, located in the eastern sector of the Bermuda triangle is populated by three groups of people; the knights, the knaves, and the normals. The knights always tell the truth; they cannot lie even to save their lives. The knaves always lie; every statement they make is false. The normals tell the truth sometimes, other times they lie.

  Many years ago[1], during the reign of queen Drumelia 1, a law was passed that knights could only marry knaves, and knaves knights, while normals could only marry normals. Thus given any married couple one of them is a knave and the other one a knight, or both are normal.

  - We first consider a married couple, Mr. and Mrs. Rumpelstiltskin, also known as Mr. and Mrs. R. They make the following statements.

    Mr. R: "My wife is not normal."

    Mrs. R: "My husband is not normal."

    What are Mr. and Mrs. R?

[1] From R. Smulyan's *What is the Name of This Book*

– Suppose instead that they had said:
  Mr. R: "My wife is normal."
  Mrs. R: "My husband is normal."

  What are Mr. and Mrs. R in this case?

– We now consider two married couples; Mr. and Mrs. Munchausen (also known as Mr. and Mrs. M) and Mr. and Mrs. Struwelpeter (Mr. and Mrs. S). Here are their statements.

  Mr. M: "Mr. S. is a knight."

  Mrs. M: "My husband is right; Mr. S. is a knight."

  Mrs. S: "That's right, my husband is a knight."

  What are each of the four people and which of the three statements are true.

Today's Problems

(10/3/15)
Rules:

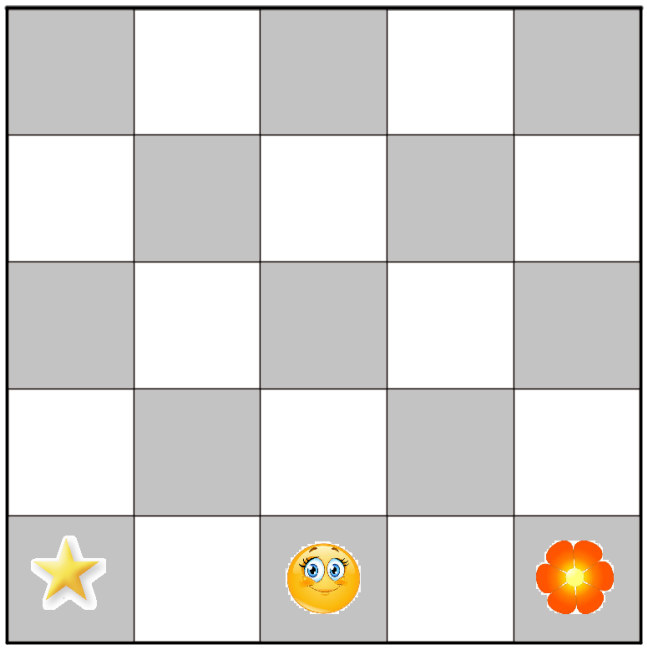
- Work on these problems in any order. You will have until about 3:30 for this activity.
- Work alone or in groups.
- **This is NOT an exam. If you have questions, need hints, just ask one of the organizers.** We want to challenge you, not frustrate you.
- Feel free to get up, walk around the room, write on the white boards with the provided markers.
- At 3:30PM, more or less I will ask for solutions, and we will discuss the solutions. Students or groups who have found solutions, time permitting, can present them on one of the white boards.

1. 201 identically shaped gears are in a row. Each gear, except the first and the last, is connected to its left and right neighbor.

   If the leftmost gear is turned clockwise ↻, is the rightmost gear going to rotate clock or counterclockwise?

2. Suppose that the 201 gears are placed in a circle in such a way that every gear is connected to two neighbors. One of the gears is turned clockwise. What happens then?

3. A checker is placed on the bottom left corner of a 5×5 checkerboard. The checker is supposed to tour the board, visiting each square exactly once. But, each time it moves, it can only move one square to the left, or one square to the right, or one square up, or one square down.

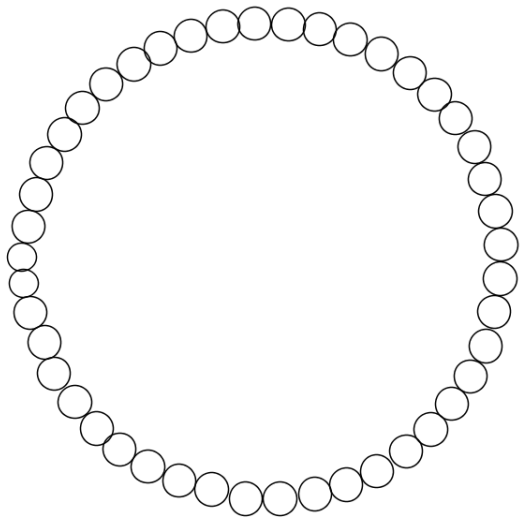


   (a) Can the checker end at the square marked with the flower? (For each question, either indicate a tour or explain why there is none).

   (b) Can the checker end at the square marked with the smiley face?

   (c) Can the checker end back at the square with the star (so visiting that square twice, in a way)?

4. Fortyfive (45) little circles are placed in a circle, as shown below.



They are to be painted using two colors, some circles are to be painted blue, others red.

(a) Prove that there must be two adjacent circles having the same color (adjacent=next to each other).

(b) Prove that somewhere we must have two circles of the same color separated by exactly two circles.

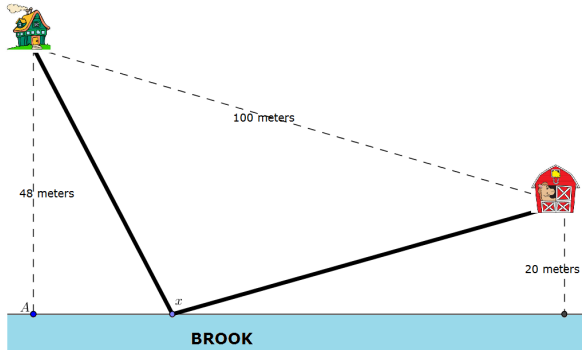
5. (a) How does the area of a rectangle change if one of its sides increases by 10% while the other decreases by 10%? (b) Same question for the perimeter.

6. A ship is on its way from one of the Caribbean islands to Miami. Halfway through the trip the ship gets a hurricane warning so it doubles its speed and arrives at Miami 3 hours ahead of schedule. How long was the whole trip.

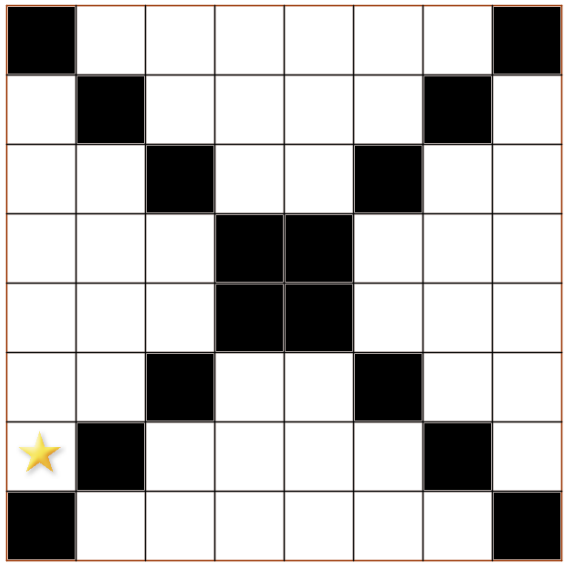
7. A passenger fell asleep on a train when the train was halfway to his destination. When he woke up, he noticed that he had half as far to go as he went during the time he slept. For how much of the journey did he sleep?

8. A notepad costs a whole number of cents. If 10 notepads cost more than \$11 and 9 notepads cost less than \$10, how much does a notepad cost.

9. Every day Jack, who lives in a house 48 meters away from a brook has to go from his house to the brook to fetch a pail of water and bring it to his horse, which he keeps in a barn 100 meters away from his house. The barn is 20 meters away from the brook. Jack wants to go in a straight line to the brook, and from there straight to the barn, following a path like the one shown in the picture. In the picture x marks the spot where Jack gets to the brook. How far away from A should x be so that Jack has to walk as little as possible?



10. A star has been placed in one of the white squares of the board, as shown. Place (or draw) 7 more stars in another 7 white squares, so that no two star are in line horizontally, vertically or diagonally.



11. The midpoints of the sides of a triangle have been marked. Then the triangle is erased leaving only the marked points? How can the triangle be recreated using only a compass and a straightedge (and a pencil)?

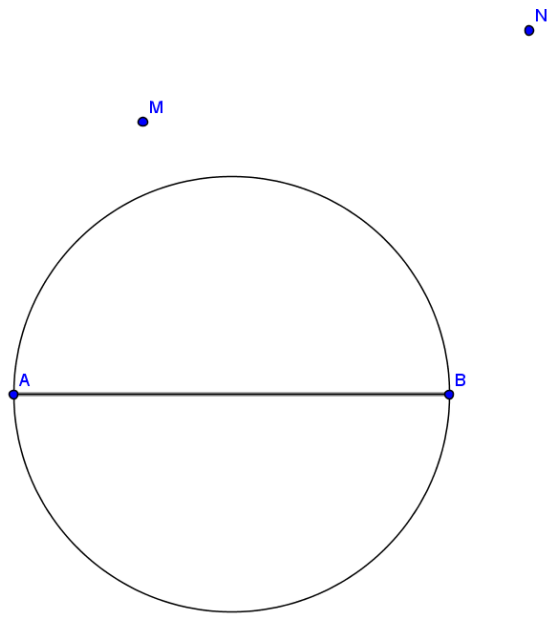
    Notice: Using only a pencil, a compass, and a straightedge means that you are ONLY allowed to do the following:

    - You may draw a line through any two points (or join them by a segment).
    - You may use any two points to draw a circle as follows: You can use one point as center and use the distance between the points as the radius; that is, you can put the pointy pinchy part of the compass on one point, open until the pencil of the compass is at the other point, and then you can draw the circle.
    - If two lines intersect, or if a line intersects a circle, or if two circles intersect, you may mark the intersection points and use them to draw more lines and/or circles.

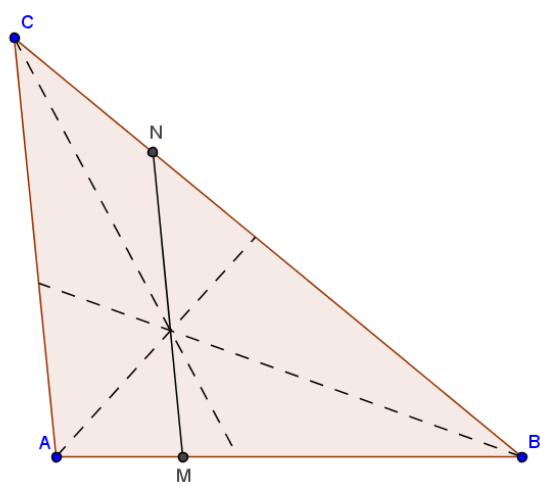
      Notice: Using only a pencil and a straightedge means that you are ONLY allowed to do the following:

      – You may draw a line through any two points (or join them by a segment).
      – If two lines intersect, or if a line intersects the circle, you may mark the intersection points and use them to draw more lines.

12. Suppose we have points M and N and a circle of diameter AB, positioned in the plane as in the diagram. How can you use only a pencil and a straightedge to drop a perpendicular from M to AB and from N to the continuation of AB?



13. In triangle ABC a line was drawn through the intersection of all three angle bisectors, parallel to side AC. This line intersects sides AB and BC in M and N respectively.



Prove that $|AM| + |CN| = |MN|$.