

# FAU Math Circle

9/26/2015

## Math Warm Up

For today's warm up we'll be doing a bit of geometry. We start with a puzzle due to one of the most famous puzzle makers of all time, Henry Dudeney (1857-1930); and it is his most famous puzzle known as the haberdasher's puzzle. He published it eventually in a collection of puzzles known as the *The Canterbury Puzzles*<sup>1</sup>. Here it is:

With three straight cuts, cut an equilateral triangle into 4 pieces and then reassemble these pieces to form a square.

This is quite hard; hard enough that the British Royal Society invited Dudeney in 1905 to show them his solution. I made things simple; I made the cuts. I am providing you with four pieces of a jigsaw style puzzle that can be assembled either to form an equilateral triangle or a square. It is your mission to assemble them as a square, and as an equilateral triangle.

The next part of the warmup will consist of some very basic geometric constructions. Maybe you know them already. Later on we will do harder ones. These constructions follow a simple rule: You can only use a straightedge and a compass.

- Given a segment, draw the perpendicular bisector.
- Given an angle, draw the bisector.

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<sup>1</sup>This and other Dudeney puzzles are available in e-book form at [www.gutenberg.org/files/27635/27635-h/27635-h.htm](http://www.gutenberg.org/files/27635/27635-h/27635-h.htm)

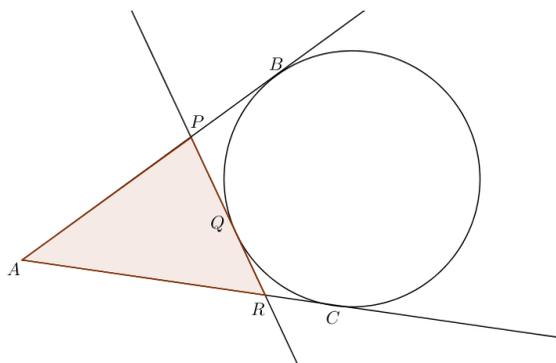
## Today's Problems

### Rules:

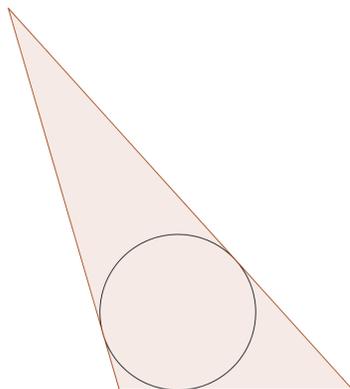
- Work on these problems in any order. You will have until about 3:30 for this activity.
- Work alone or in groups.
- Feel free to get up, walk around the room, write on the white boards with the provided markers.
- At 3:30PM, more or less I will ask for solutions, and we will discuss the solutions. Students or groups who have found solutions, time permitting, can present them on one of the white boards.

1. Let us recall the last problem from the last session. Let us see if we remember how it was solved by Callie.

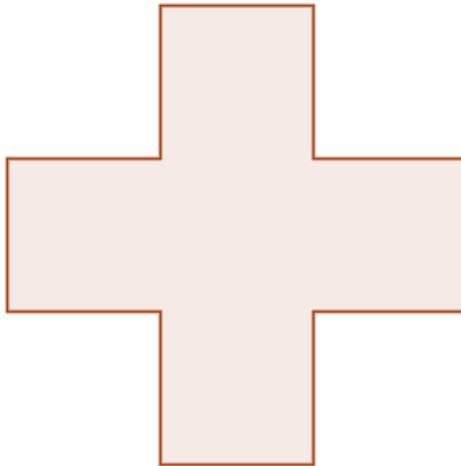
From a point  $A$  outside of a circle we draw two tangents touching the circle at points  $B, C$ , respectively. We then draw a third tangent intersecting segment  $AB$  at  $P$ , segment  $AC$  at  $R$  and touching the circle at  $Q$ . If  $|AB| = 20$ , what is the perimeter of triangle  $APR$ ? Can one even determine it from the provided data?



2. The ideas in the solution of the previous problem play a role here. The perimeter of a triangle is 75 inches, the radius of the inscribed circle is 5 inches. What is the area of the triangle?



3. One of the legs of a **right** triangle has length 12 and the hypotenuse has length 13. Find the area of the inscribed circle. This should be quite easy after having done the previous problem.
4. A triangle has sides of length 9, 10, and 17. What is the radius of the inscribed circle? This can be easy if we know Heron's formula. If not, it could be hard.
5. Using only a compass and a straightedge, how can one find the *incenter* of a triangle; the center of the inscribed circle? On page 4 of this problem set you'll find a few triangles on which you can practice. The incenter must be at the same distance from each one of the sides; is there always such a point?
6. The cross pictured below, having all arms of the same length, can be divided by two cuts into four pieces that can be assembled to form a square.



On the page 5 you will find several copies of the cross to practice your cuts.

7. Every day Mr. Wooster returns from his club in the city (where he spends the day drinking tea and gossiping) by train to the train station of the town where he lives. His butler Jeeves is supposed to pick him up by car and drive him home. No time is to be wasted; Jeeves is to leave the house so he exactly arrives at the train station the moment the train pulls in, Mr. Wooster jumps into the car and they drive immediately home. One day the train arrives early, Jeeves isn't there yet, and Mr. Wooster decides to walk home. After walking for half an hour he meets Jeeves on the way to pick him up. He gets into the car and they arrive at the home 20 minutes earlier than usual. How many minutes early was the train?
8. There are 2015 representatives of four of the Middle-Earth races sitting around a round table, dwarves, elves, gnomes, and humans. Elves never sit next to dwarves, and humans cannot sit next to gnomes. Prove that at least one pair of representatives sitting next to each other must be of the same race.
9. Suppose there are exactly 9 towns in a very small country and **all distances between the towns are different**. A person starts in each town and walks towards the **nearest** town. Prove: (a) There are two towns A and B such that a person from A walks to B, and a person from B walks to A. (b) There is a town that nobody walks to.
10. A certain country has several airfields with lots of planes in each. The distances between all of the airfields are different; no two are at the same distance. One day an airplane takes off from each field and lands on the closest airfield. Prove that at most 5 airplanes will land on each airfield.

# TRIANGLES AND CROSSES

