On Unimodality of Independence Polynomials of Trees

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An independent set in a graph is a set of pairwise non-adjacent vertices. The independence number $\alpha(G)$ is the size of a maximum independent set in the graph G. The independence polynomial of a graph is the generating function for the sequence of numbers of independent sets of each size. In other words, the k-th coefficient of the independence polynomial equals the number of independent sets comprised of k vertices. For instance, the degree of the independence polynomial of the graph G is equal to $\alpha(G)$. In 1987, Alavi, Malde, Schwenk, and Erdös conjectured that the independence polynomial of a tree is unimodal. In what follows, we provide support to this assertion considering trees with up to 20 vertices. Moreover, we show that the corresponding independence polynomials are log-concave and, consequently, unimodal. The algorithm computing the independence polynomial of a given tree makes use of a database of non-isomorphic unlabeled trees to prevent repeated computations.

Keywords: independent set, independence polynomial, tree, log-concave sequence, unimodal sequence.