Counting in Graphs: An Invitation to Graph Polynomials

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Many graph polynomials can be considered as generating functions for certain types of (spanning, induced, general) subgraphs of a given undirected graph. In this talk, we consider common techniques of computation that can be applied to a large class of graph polynomials, where we restrict to graph polynomials that are invariant with respect to graph isomorphism. Typical examples are chromatic, flow, reliability, independence, cycle, and domination polynomials. More generally, we can define a graph polynomial f by

$$f(G; x_1, \ldots, x_r) = \sum_{H \leq G} [p(H)] \prod_{i=1}^{r} x_i^{k_i(H)},$$

where $H \leq G$ means that H is a (spanning, induced) subgraph of G, p is graph property (like connected or is cycle-free), and $k_1, \ldots, k_r$ are graph functions (for instance the number of components, vertices, edges).
We will consider the following questions:

- How can we calculate a graph polynomial?
- What is the computational complexity?
- Which kind of information about the structure of a graph can we extract from the polynomial?
- How can we compare graph polynomials?

It is clear that this presentation cannot be comprehensive. It is only an attempt to draw the attention of the audience to the fascinating subject by presenting some nice results and applications as well as some sufficiently difficult open problems.

Keywords: graph polynomials, enumeration in graphs