Paths are generically realisable

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Building on the idea of Strong Spectral Property (SSP), we generalise the notion of the nowhere-zero eigenbasis in the following way. We show that given a connected graph $G$ of order $m$, a finite set $\mathcal{Y} \subset \mathbb{R}^m \setminus \{0\}$, and a diagonal matrix $\Lambda$ with distinct real diagonal elements, there exist matrices $A$ and $U$ so that $A$ has the SSP and associated graph $G$, $U$ is orthogonal, $U^T A U = \Lambda$ and $Uv$ has no zero entries for all $v \in \mathcal{Y}$. We explore some applications of this result to the Inverse Eigenvalue Problem for Graphs.

Keywords: Inverse eigenvalue problem for graphs; Strong Spectral Property; Generic Realisability.