Fashion game on graphs

Chenli Shen, Wensong Lin, Southeast University

In this paper, we propose and study an optimization problem of the fashion game on graphs, which can be regarded as a graph extension of matching pennies. There are two kinds of players in a graph $G$: Conformists and Rebels. All players choose their actions from an identical set of the two symmetric actions, say $\{0, 1\}$. An action profile $\pi$ of $G$ is a mapping from the vertex set of $G$ to the action set $\{0, 1\}$. A conformist (resp. rebel) likes people having the same (resp. different) action with her and dislikes people having the different (resp. same) action. The utility $u(v, \pi)$ of a player $v$ under the action profile $\pi$ is the number of neighbors she likes minus the number of neighbors she dislikes. The utility $u(G, \pi)$ of $G$ under $\pi$ is the minimum utility among all players. Let $t$ be an integer. A graph $G$ is said to be $t$-satisfiable if there is an action profile of $G$ such that all players have utilities at least $t$. The utility of $G$, denoted by $u(G)$, is the maximum $t$ such that $G$ is $t$-satisfiable.

We provide simple characterizations to determine the utilities of paths, cycles, and complete graphs. We design a linear-time algorithm to determine the utility of a tree. We obtain lower bounds of utilities of graphs containing only rebels, $t$-degenerate graphs, and the $k$th power of paths, respectively. We also prove that for any fixed integer $t \geq -2$, the problem of deciding if a graph containing both conformists and rebels is $t$-satisfiable is NP-complete, and for any fixed integer $t \geq 1$, the problem of deciding if a graph containing only rebels is $t$-satisfiable is also NP-complete. We finally propose some further research problems on this topic.

Keywords: Fashion game, Conformists, Rebels, Utility, $t$-satisfiability problem, Defective coloring.