Inequalities Connecting the Annihilating and Independence Numbers

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Given a graph G, the number of its vertices is $n(G)$, while the number of its edges is $m(G)$. An *independent set* in a graph has no vertices adjacent to each other, and $\alpha(G)$ is the size of a maximum independent set. A set of edges is a *matching* if no two of them have a vertex in common, and $\mu(G)$ is the size of a maximum matching. If $\alpha(G) + \mu(G) = n(G)$, then G is a *König-Egerváry graph.*
If $d_1 \leq d_2 \leq \cdots \leq d_n$ is the degree sequence of G, then the *annihilation number* $a\,(G)$ is the largest integer k such that $\sum\limits_{i=1}^{k} d_i \leq m(G)$ (Pepper, 2004).
It is known that $\alpha(G) \leq a(G)$ for every graph G. We intend to estimate the difference between these two parameters. More specifically, we prove a series of inequalities, including $a(G) - \alpha(G) \leq \frac{\mu(G)-1}{2}$ for trees, and $a(G) - \alpha(G) \leq 2 + \mu(G) - 2\sqrt{1 + \mu(G)}$ for bipartite graphs. Moreover, we show that these inequalities present tight upper bounds on the difference between the annihilation number and the independence number, no matter what value is assigned to $\mu(G)$. In addition, we discuss the corresponding estimations for König-Egerváry, triangle-free, and general graphs.

Keywords: annihilation number, independence number, tree, bipartite graph, König-Egerváry graph, triangle-free graph