The Target Pebbling Conjecture

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Introduced by Chung in 1989, graph pebbling is a network optimization model for satisfying vertex demands with vertex supplies (called pebbles), with partial loss of pebbles in transit. A *configuration* C of *size* $|C| = p$ on a graph G is a supply of p pebbles on the vertices of G. Similarly, a *target distribution* D of *size* $|D| = t$ on G is a demand of t pebbles on the vertices of G. A *pebbling step* from u to v removes two pebbles from u and places one of those pebbles on v; the other pebble vanishes as a toll. We say that C *solves* D if C can be converted via pebbling steps to a configuration C^* such that $C^*(v) \geq D(v)$ for each vertex v. Generalized in 2005 for $t > 1$ by Crull, et al., the *pebbling number*, $\pi(G, D)$, of the target distribution D is defined to be the smallest m such that every configuration of size m solves D. Write r^t for the target distribution consisting of t pebbles on vertex r, and set $\pi_t(G) = \max_{r \in V} \pi(G, r^t)$. In 2013, Herscovici, et al., proposed the *Target Conjecture*, that every graph G satisfies $\pi(G, D) \leq \pi_{|D|}(G)$ for every target distribution D. They verified the conjecture for all D when G is a tree, cycle, complete graph, or cube. Earlier work of Sjöstrand (2005) verifies the conjecture for all G when $D(v) = 1$ for each vertex v. In this talk we discuss recent verifications of the Target Conjecture for other classes of graphs, and introduce a *Strong Target Conjecture* and related results.