

An Introduction to Parking Functions

Kimberly P. Hadaway*, Dr. Pamela E. Harris, Williams College

In 1966, Alan G. Konheim and Benjamin Weiss defined “parking functions” as follows: We have a one-way, one-lane street with n parking spaces, numbered in consecutive order from 1 to n , and we have n cars in line waiting to park. Each driver has a favorite (not necessarily distinct) parking spot, which we call its *preference*. We order these preferences in a *preference vector*. As each car parks, it drives to its preferred spot. If that spot is open, the car parks there; if not, it parks in the next available spot. If a preference vector allows all cars to park, we call it a *parking function*. In 1974, Henry O. Pollak proved the total number of parking functions of length n , meaning there are n parking spots and n cars, to be $(n+1)^{(n-1)}$. In this presentation, we describe a recursive formula, expound Pollak's succinct six-sentence proof of an explicit formula, and conclude with a discussion of other parking function generalizations.

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