Complexity Class Boundaries and Incremental Changes to the Birkhoff Polytope

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This paper proposes the idea of developing incremental changes to decision problems, together with some kind of corresponding set of minimal incremental changes to polyhedra in an attempt to correlate problems, polyhedral structure and complexity class boundaries. The hope is to apply this idea to distinct complexity classes in search of articulating complexity class boundaries. As an example, assuming **P**≠**NP**, and Graph Isomorphism (GI) is neither in **P** nor **NP-complete**, a **P** problem is incrementally modified to become GI through to an **NP-complete** problem. An IP is implemented whereby the feasible region of the **P** problem is the Birkhoff polytope, decidable in polynomial time. The IP is evolved via incremental modifications made to the Birkhoff polytope, until it becomes an **NP-complete** problem. Separate from these evolutions, and by assumption, the complexity of algorithms that decide intermediate problems can't help but likewise evolve from polynomial through to *something greater*, having started as a polynomial time algorithm. The goal then is to incrementally develop an easy problem into a hard problem by design so that corresponding intermediate problem algorithm complexities more closely track intermediate problem complexity classifications, if possible, whatever they may be. How these complexities are intentionally evolved is then of great interest i.e. a problem's complexity classification is distinct from the complexity of an algorithm that decides the problem (although bounded).

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