Powers of Two as Sums of Two Balancing Numbers

Jeremiah Bartz*, Bruce Dearden, Joel Iiams, Julia Peterson, University of North Dakota

Let $(B_n)_{n\geq 1}$ be the sequence of balancing numbers given by $B_1 = 1$, $B_2 = 6$, and $B_n = 6B_{n-1} - B_{n-2}$ for $n \geq 3$. In this talk, we consider the exponential Diophantine equations $B_n = 2^a$ and $B_n + B_m = 2^a$. Using Baker's theory of logarithmic forms, Matveev's Theorem, and an additional reduction theorem, we establish bounds on the space of possible solutions. This remaining space is sufficiently small that the problem of identifying solutions is reduced to a computational search which is carried out by a simple computer program.

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