Bounds on the (Edge)-Fault-Diameter of Bipartite C_4-free Graphs

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The distance between two vertices u and v in a connected graph G is the length of a shortest $u-v$ path in G. The diameter of G is the largest of the distances between all pairs of vertices of G. If the removal of not more than k vertices (edges) never disconnects the graph G, we say that G is $(k+1)$-connected ($(k+1)$-edge-connected). The k-fault diameter and k-edge-fault diameter of a $(k+1)$-connected or $(k+1)$-edge connected graph G is the largest diameter of the subgraphs obtained from G by removing up to k vertices and edges respectively.
Few bounds on the fault-(edge)-diameter are known. Recently, the second author [Bounds on the fault-diameter of graphs, Networks 70(2) (2017), 132-140] observed that the k-fault diameter of a $(k+1)$-connected graph G with n vertices is bounded from above by $n-k+1$ and showed that this bound can be improved to approximately $\frac{4n}{k+2}$ if G is triangle-free and $\frac{5n}{(k-1)^2}$ if G does not contain 4-cycles. He also gave similar bounds on the k-edge-fault-diameter.
In this talk, we present these results and show that the above bound for C_4-free graphs can be improved to $\frac{3n}{k^2-k+1}+3k^2-3k+5$ if, in addition, the graph is also bipartite. We also discuss results on the k-edge-fault diameter for $(k+1)$-edge connected C_4-free and bipartite C_4-free graphs. We construct graphs to show that our bounds on the k-fault-(edge)-diameter of bipartite C_4-free graphs are best possible.

Keywords: diameter; fault-diameter; edge-fault-diameter; minimum degree