Product throttling for the game of Cops and Robbers

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Cops and Robbers is a graph game in which some number of cops and a robber alternate turns moving along the edges of a graph. The cops win if a cop moves to the vertex where the robber is. The capture time for \( k \) cops on a graph \( G \), denoted \( \text{capt}_k(G) \), is the minimum number of turns it takes \( k \) cops to catch the robber. Throttling refers to the concept of balancing time and resources spent during a graph game, this was first introduced for the zero forcing process. Traditional (sum) throttling involves the summation of time and resources, we introduce product throttling for the game of Cops and Robbers as a more realistic measure of the man-hour cost of pursuit.

Product throttling is defined to be \( \text{th}_x(G) = \min_k (k(1+\text{capt}_k(G))) \), this measures the minimum cost of catching the robber assuming each cop is paid one dollar to be placed on the graph and then another dollar for each round of the game. Results to be presented include general bounds on product throttling in terms of sum throttling, a characterization of graphs with small product throttling number, and the value of product throttling for chordal graphs in terms of their radius.

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