The Very Strong Proper Connection Number and Shortest Path Colorings

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A proper edge coloring of a graph is an edge coloring such that no two incident edges in the graph share the same color. The proper connection of a graph is a variation of proper edge coloring introduced by Borozan, et al. According to their definition, an edge colored graph is said to be properly connected if there exists a properly colored path between every pair of vertices. They define the proper connection number of a graph, denoted \(pc(G)\), as the minimum number of colors, \(k\), such that there exists a \(k\) edge coloring of \(G\) which is properly connected. Motivated by the work on proper connection, my Summer 2019 REU group and I extended this idea by requiring that every shortest path between each pair of vertices be properly colored. We call such a coloring a very strong shortest path coloring. Furthermore, we define the very strong proper connection number of a graph, denoted \(vs pc(G)\), as the minimum number of colors required to edge color a graph \(G\) such that the coloring is a very strong shortest path coloring. In this talk, I will discuss our results on the very strong proper connection number for certain classes of graphs.

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