Regular graph and some vertex-deleted subgraph

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In this paper, we consider a relationship between a regular graph and an regular factor of its vertex-deleted subgraph. Katerinis [Regular factors in vertex-deleted subgraphs of regular graphs, *Discrete Math.* 131 (1994) 357–361] proved that if $r$ is even integer and $k$ is integer with $1 \leq k \leq \frac{r}{2}$, and $G$ is an $r$-regular, $r$-edge-connected graph of odd order, then $G - x$ has a $k$-factor for each $x \in V(G)$. When the result “for each $x \in V(G)$” of Katerinis is replaced “for some $x \in V(G)$”, we consider what condition can be followed. One of our main results is that let $r$ be an integer with $r \geq 4$, and let $G$ be an $r$-regular, 2-edge-connected graph. If $G$ is not bipartite, then there is some $x \in V(G)$ such that $G - x$ has a 2-factor. Another of our main results is that let $r$ and $k$ be an even integer such that $4 \leq k \leq \frac{r}{2}$, and $\ell$ be a minimum integer such that $\ell \geq \frac{r}{r-2k+4}$, and $G$ be an $r$-regular, $2\ell$-edge-connected graph of odd order. Then, there is some $x \in V(G)$ such that $G - x$ has a $k$-factor. Moreover, if $r \geq 4k - 8$, then we can replace $2\ell$-edge-connected with 2-edge-connected.

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