Weighting the Edges of a Connected Graph to Achieve Metric Dimension One

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A resolving set in a metric space \((X,d)\) is a subset \(A\) of \(X\) such that for any pair \((x,y)\) of different elements of \(X\) there is some element \(a\) of \(A\) such that \(d(a,x)\) is not equal to \(d(a,y)\). The metric dimension of \((X,d)\) is the minimum cardinality of a resolving set in \((X,d)\).

If the edges of a connected graph \(G\) are assigned positive weights by a weighting function \(w_t\), a metric space \((V(G), d)\) comes into existence in an obvious way. It is easy to see that \(w_t\) can be found so that this metric space has metric dimension 1. We consider two optimization problems: Given \(G\), find \(w_t\), taking values in the positive integers, so that the resulting metric space has metric dimension 1, and (a) the maximum value taken by \(w_t\) is as small as possible; or (b) the sum of the weights assigned by \(w_t\) is as small as possible. We solve these problems for some classes of graphs and give a few general bounds.