The Hamilton-Waterloo Problem with $C_6$ and $C_{3x}$ factors

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A solution of The Hamilton-Waterloo Problem, that is, a resolvable $(C_m, C_n)$-decomposition of $K_v$ into $rC_m$-factors and $sC_n$-factors is denoted by $(m, n) - \text{HWP}(v; r, s)$. This problem has been solved for $v \leq 17$ and for $v \leq 10$ when $v$ is even. The most difficult case is either when $r$ or $s$ is equal to 1. In this talk, I will give the construction of $(6, 9) - \text{HWP}(18; 1, 7)$, and settle the problem for $v = 18t$ when $t$ is odd. Furthermore, in order to extend the latter idea to the case $n = 3x$, we proved that there exists a $(6, 3x) - \text{HWP}(6xt; 1, 3xt - 2)$ for all odd $x \geq 3$, and that there exists a $(6, 3x) - \text{HWP}(3xt; 1, \frac{3xt-4}{2})$ for all even $x \geq 4$.

Keywords: 2-factorization, Hamilton Waterloo problem, Graph Decomposition, cycle decompositions